Intuition in Mathematics: A Way of Knowing

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The purpose of this paper is to explore intuition in mathematics as a way of knowing, beginning with an overview of the philosophical history that underpins the current perspective of intuition in mathematics. This will be followed by a discussion of the assumptions of intuition in mathematics and the implications for research. Finally, I will discuss my own perspectives of how intuition in mathematics compares to my own way of knowing.

History

By limiting the scope of this paper to an overview of intuition in mathematics, it is necessary to limit the scope of an overview of the history of intuition to those philosophers who contribute to the current understanding of intuition in mathematics as a way of knowing.

Much of what we understand of the philosophical roots of intuition as a way of knowing begins with Aristotle. He argued that to defend by reasoning you must construct a syllogism, to form an argument that will extend knowledge from what is better known and more credible to what is being defended. Following this rule we find that the highest order general truths are understood intuitively, they are immediately evident, and can be grasped, even by the general populace. There is no evidence that Aristotle found the need to defend these generalized truths (Bolton, 2014).

While Descartes (1701/1999) also uses intuition as his starting point in his search for truth and the truth of mathematics, his method is quite different from Aristotle's use of syllogism. Descartes uses intuition to describe things that are self-evident, that can be directly known: that we exist, that we think, that three straight lines bound a triangle. He argues that intuition and deduction are the only valid methods for discovering truth (Mursel, 1919). He separates intuition from deduction by describing deduction as the continuous thought in which each element is intuited creating a deductive chain in which each link is intuited and the connections between them are also intuited. His method is built on the process of reducing a problem to parts small enough to be understood intuitively. Once the understanding is established his model uses deduction to build more complex truths. In rule twelve, he makes the distinction between what is known, and the individual who knows. The individual uses a combination of intuition and deduction to discover what is unknown by comparing it to what is known. In rule three he states that scientific knowledge can only be known through what we can intuit ourselves, or deduce with certainty (Descartes, 1701/1999).

Kant differs from Descartes by locating the primary source of a priori mathematical knowledge in the senses rather than the intellect. Kant divides ways of knowing into the analytic and synthetic. Analytic truths are those that are self-evident to the senses and require no empirical confirmation. Synthetic truths are those that must be pulled together (synthesized) from various experiences to obtain knowledge. Kant argued that these synthesized truths can be known *a priori* and is evidenced in our understanding of numerical and physical concepts (Robinson, 2014). The resulting mental representation allows for understanding of the analytic truths contained within because what is universally true for the parts must be universally true for the whole (Hauser, 2015).

In the beginning of the twentieth century, Husserl's phenomenology replaces Kant's sense perception with a shift in focus from the nature of an object to the phenomenological manner in which objects present themselves to consciousness, dividing them into the real (physical) and the categorical (ideal) (Zagorianakos & Shvarts, 2014). Husserl was concerned with the conceived manner of mathematical idealities. His sequence of complex acts of consciousness allow for the knowledge of concepts void of the objects that they reference. Truth is the result of the coincidence of the meant with the given. This allows us to understand truths that are independent of our experiential knowledge. An object does not need to physically exist in order to be understood (Hauser, 2015).

In the mid-20th century, Gödel presents the perceptualist view of mathematical intuition. Perceptionalism is a form of phenomenological experience of sensoril awareness of objects in the environment, even if those objects are not perceived by the physical senses. Gödel describes the abstractness of mathematical intuition as not outside of our ability to perceive, but as a kind of sense perception in itself. In his view, mathematical sense perception is used by mathematicians to understand the objects in set theory in the same way, and with no less confidence, that sense perception is used by natural scientists to describe the natural world (Chudnoff, 2011, 2014; Hauser, 2015).

Assumptions

Intuition is as often defined by what is not, as it is by what it is. *Merriam-Webster* (2015) defines intuition as “the power or faculty of attaining to direct knowledge or cognition without evident rational thought and inference.” One way to make sense of the fundamental assumptions about the nature of intuition in mathematics is by looking at intuition through the lens of dual-process theory. This also will allow us to consider the assumption that intuition is not simply the opposite of rational thought, but is also seen as the starting point of more deliberative approaches to knowing.

Dual-process theory, or the dual-process system, is the concept that knowing is processed by two independent systems: the intuitive, and the deliberative (Epstein, 2011). The intuitive half of the dual-process system has been described as knowledge that is automatic and unconscious as opposed to the step-by-step process of deliberative reasoning (Glockner & Whitteman, 2010;

Hauser, 2015; Pretz, 2011; Sinclair, 2011). Intuition is seen as the instantaneous, holistic, non-sequential judgment of a highly experienced expert (Kidd, 2014; Klein, 2015) who is able to recognize patterns without taking time for deliberation. Intuition has also been described as an automatic inference, or reaching a conclusion based upon little information (Glockner & Whitteman, 2010; Pretz, 2011). Intuition is also used to describe the unlearned, tacit understanding of numbers that infants display (Starkey & Cooper, 1980).

If we think about intuition as one half of a dual process, it would be easy to assume that the processes are in opposition to each other. On the contrary, intuition can be seen as the catalyst that allows for the creation of the environment necessary for the rational half of the dual-process system to function. In this light, it is reasonable to assume that the philosophers that we have examined would consider a fundamental assumption of intuition to be that it is foundational to further inquiry. Aristotle considered intuition to be the source of the most general truths used to explain less general truths through the inductive reasoning process (Bolton, 2014). Descartes chain of reasoning is forged from elements that have been broken down into parts that are small enough to be understood intuitively (Descartes, 2003). Kant argues that we need non-empirical intuition of mathematics and physics in order to construct a universal representation of concepts (Friedman, 2012: Zagorianakos & Shvarts, 2014). Husserl’s immediate intuition of an object is used as the basis for understanding the object’s application in different situations (Hauser, 2015). And, Gödel understood mathematical intuition as a kind of sense perception that perceives mathematical objects as the foundational building blocks of mathematical theory formation.

Implications for Research

The challenge for developing research in mathematics intuition is in the operationalization of the concept of intuition (Sinclair, 2011). Here again dual-process theory gives us the vocabulary to develop constructs that can be studied (Epstein, 2011).

Beginning at the beginning of life, research has been conducted to assess the question: How much mathematical intuition we are born with, and does this innate understanding predict the later acquisition of mathematical concepts? In a study of 72 infants with a mean age of 22 weeks (Starkey & Cooper, 1980), the infants were able to discriminate between small amounts, recognize representations of numbers, and demonstrate the ability to remember quantity. The authors of this early study question whether there is a possible link between these abilities and the development of verbal counting. A recent longitudinal study (Starr, Libertus, & Brannon, 2013) explores the question: Is the intuitive number sense that infants display the basis for learning to count and acquire symbolic mathematical knowledge? The study followed 48 3.5-year-old-children who had been tested at six months of age on numerical change detection and measures of intuitive number sense, and found that their scores at six months significantly predicted their standardized test scores at 3.5 years of age, including the understanding of verbal counting principles.

Looking explicitly at intuition's role in dual-process theory allows us to ask several questions. Are there are interactions between the intuitive process and the rational processes? Is there a relationship between a student's employment of either method during problem solving and achievement outcomes? How does the nature of a problem affect students' problem solving processes? In a study that the authors describe as the first application of dual-process theory to mathematics education (Leron & Hazzan, 2006), the authors review several prior studies that gave college level computer science and mathematics students either simple representations of a problem or a more complex construction of the same problem, and compared the results. The studies found that often students who had been given the more complex construction of the problem had better outcomes than the students who were given the simple representation. The researchers suggest that the reason that this outcome is so common is that the more complex construction blocked the intuitive problem solving process in the students and forced them to work more deliberately. In a later study (Leron & Paz, 2014), the question of whether intuitive thinking helps or hinders analytical thinking is further explored. Here too the researchers found that the more difficult version of the task elicited a significantly higher-level outcome from the participants. The authors again conclude that the students scored better on the more difficult instructions because the messier input suppressed the intuitive system. In an effort to bridge the gap between intuition and rational knowledge, a study was conducted (Yavuz, 2015) to test if college level students' trusted in mathematics calculation or intuition to predict an event in a problem solving activity. This study found that students' perceived trust in intuitive knowledge was higher than their confidence in mathematical conclusions, though incorrect mathematical conclusions were often selected over correct intuitive predictions. The authors suggest that a combined approach of intuitive and quantitative problem solving approaches may be preferable.

While the greater part of this exploration of intuition in mathematics has addressed intuition's contribution to rational thought processes, the gap between the two processes may be bridged from the other direction. Specifically: In what way does the deliberative, rational process contribute to the intuitive process? How does knowledge and experience move from acquisition to intuitive sense knowledge? An intriguing study was conducted to determine if procedural understanding in grade school students manifested in intuitive mathematics understanding in adults (Varma & Schwartz, 2011). In the study, both adults and sixth grade children were tested on their understanding of positive and negative integers. While the children were able to compute the correct numerical answers to questions about the quantitative difference between integers, they did not display an understanding of the relative distance between the integers. On the other hand, adults were able to demonstrate an understanding of the distance as a higher order concrete representation between the integers. While the children used rules to solve the problems, the adults displayed an intuitive understanding of the concrete meaning behind the numbers. The authors hypothesize that adults have restructured symbolic rules into an intuitive understanding of the abstract mathematical concept.

Discussion

I chose to research intuition as a way of knowing out of curiosity. I was hoping to gain not only a greater understanding of intuition, but also a greater insight into my own way of knowing. I came to the subject with only the common understanding of intuition as that unknown something that we understand without knowing why we understand it. I have learned that intuition in mathematics encompasses this understanding and refines how this process occurs in different populations. In addition, by looking at the difference between intuition as a way of knowing, and deliberative reasoning processes, we can form questions appropriate for research.

To make a cogent analysis of the difference between mathematics intuition as a way of knowing and my own way of knowing mathematics, it is necessary to begin by describing how I come to know mathematical concepts. It would seem that after thirty years of teaching mathematics to grade school and undergraduate students that there would be little new knowledge for me to acquire. The constant development of new technologies, and the mathematical reasoning abilities necessary to engage with them requires me to constantly update and revisit my mathematical skill set. Describing how I come to know new processes and concepts is particularly challenging as language is by nature linear, and how I process mathematical knowledge is global. Mathematical procedures, like language, are linear in form. In the same way that an algebra student works through the steps of an equation, I too step through problems according to predetermined proved mathematical procedures. But how I know is an integration of rational procedural processes and intuitive sense making, not sequentially, but in tandem. In a metacognitive way, I observe myself following the procedures that mathematics provides while my intuitive sense anticipates and marvels at the orderly process that reveals truth. Gödel's perceptionalism describes quite accurately my experience of logic and intuition functioning in unison when I explore mathematics, similar to the way that our sense of smell and taste work when we experience the flavor of a favorite dish. I think another accurate analogy is the flow state that athletes experience when they are simultaneously immersed in decision making and high level skill performance.

This contrasts with the either/or view of intuition's function in the dual-process model. Outside of Gödel, the philosophers and researchers earlier discussed in this paper, separate the processes. They discuss the appropriateness of the application of intuition or deliberative rational process according to the problem presented. The philosophers do not discuss intuition functioning during the deliberative process. The researchers approach intuition and reason as entities that either act in sequence, or react by displacement. While one process may inform the other, they are not seen as functioning simultaneously.

Perhaps the difference between my experience and the philosophical underpinnings and research models of mathematical intuition, is simply a demonstration of the difference between human experience and how that experience must be defined in order to be studied.

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