Nonverbal and Textual Use in High School Geometry Textbooks:

Volume of Geometric Solids

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Toya Frank: Heading: Introduction

The use of imagery in textbooks can be powerful (Levin & Mayer, 1993), so it was no surprise that there have been many studies on the relationship between images and text in mathematic textbooks. Studies have included research on the interaction between nonverbal elements and the reader (Anglin, Towers & Levie, 2004) and the effect of the juxtaposition of text and imagery on cognitive load (Sonnenfeld & Keebler, 2016).

Mayer, Sims and Tajika (1995) conducted a compelling proportional analysis of the use of text in their comparison of textbooks from the United States and Japan. Their research supported their hypothesis that the allocation of space within a textbook reflected the curriculum goals of the culture in which they were created. I was unable to find research that compared the proportional use of space within a mathematics textbook analyzed by how the elements are used within the textbook. Their study included a proportional comparison of the space used on each page for exercises, irrelevant and relevant illustrations, and explanations.

The purpose of this study was to design a quantitative methodology that would allow me to conduct a comparative analysis of the verbal and nonverbal elements of three high school geometry textbooks that represented three different approaches to mathematics pedagogy: New Math developed and implemented in the 1960s and mostly out of use (Miller, 1990), traditional mathematics instruction still in use, and reform math aligned with NCTM standards and currently in use (Serra, 2008, p. iv).

Toya Frank: Heading: Relevant Literature ???

Toya Frank: How do these pedagogical approaches connect to what you are doing in this paper. Make your connections clear.

Pedagogical Approaches

Traditional mathematics instruction is characterized by didactic representation of the material followed by worked examples. The emphasis is on procedural knowledge and attaing correct answers to problems with singular answers. Exploration beyond procedures usually includes word problems that are not necessarily grounded in the possible experiences of the learner (Vilardi & Rice, 2014).

New Math was a pedagogical approach developed during the cold war to address the rising need for college bound students with solid math skills. The intent was to guide students into the discovery of abstract mathematical principles by deduction (Miller, 1990).

Discovery Learning was aligned with the NCTM Standards (Serra, 2008, p. iv) which include content standards of understand numbers and operations, the application of algebraic symbols and procedures, understanding geometry by characterizing shapes and relationships, and understanding the attributes, techniques, tools, and formulas used to determine measurement (NCTM, 2000).

Geometric Topic

The topic I chose to review across the textbooks was the measurement of volume of three-dimensional geometric forms. I chose this topic because of its importance in the preparation for college level mathematics, the well-researched difficulty that secondary students have with the topic (Battista & Clements, 1998; Sisman & Aksu, 2016). and because there was very little research available on the understanding of volume in high school mathematics students (Dorko & Speer, 2013).

Overview of Textbooks

Before I began the analysis with the three chosen textbooks, I flipped through the pages of each to get a general sense of the textbooks' look and feel. What struck me the most was how similar the illustrations were in spite of [redacted]-year timespan that the texts represented. In spite of the differences, all three texts use many of the same conventions that Euclid used (Euclid,

Toya Frank: interesting.

Toya Frank: powerful insight and opinion.

2013; Euclid's Elements, n.d.; Figure 1), reminding me that the last 30 years are nothing to the 2000 years of mathematics pedagogy that, depending on one's perspective, either holds back the forward progress of mathematics pedagogy, or continues to provides a solid dependable foundation of knowledge that was still useful to mathematics educators.

Figure 1

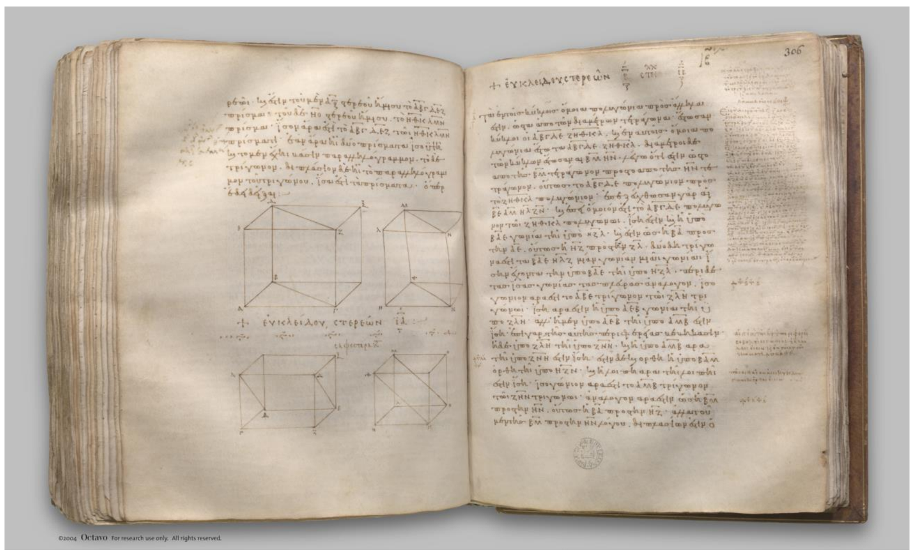


Figure 1. Proof of the volume of a right rectangular prism from Euclid's Elements, Book XII. (n.d.). *In Clay Mathematics Historical Archive*. Retrieved from http://www.claymath.org/library/historical/euclid/book12.html

Purpose and Research Questions

There were two man purposes for this study. The first was to extend Mayer, Sims and Tajika's (1995) research by developing a framework of that applies their categories of text usage (exercises and explanations) to nonverbal elements, and their categories of relevance to both nonverbal and textual elements. The second purpose was to use the developed framework to identify the proportion of the quantity and type of use of nonverbal and textual elements of textbooks drawn from three different pedagogical traditions.

The following research questions were explored:

- What was the proportion of non-verbal content vs. text in high school geometry textbook units on volume measurement of cylinder, prisms, cones, and pyramids?
- How do textbooks with very different philosophical pedagogical perspectives use of nonverbal elements vary?

Materials and Methods

Data Sources: Textbooks Chosen

The first and oldest textbook selected for analysis was *Geometry, Student's Text Part II Unit 14* prepared under the supervision of the Panel on Sample Textbooks for the School Mathematics Study Group and published by Yale University Press in 1961 (Allen et al., 1961). The "SMSG program emphasizes the objectives of inquiry and discovery,"(Gall, 1970, p. 713). The pages are typewritten, with hand-drawn illustrations and diagrams. The topic was introduced and explained with geometric diagrams, equations, and text describing vocabulary meanings and theorem proofs. The explanations are followed by a numbered set of exercises that are a mix of abstract and real-world based problems.

The second text reviewed was the Larson, Boswell, and Stiff (2004) *McDougal Littell Geometry* textbook: a traditional textbook that has been widely used in the United States. Each topic was presented in discrete units, beginning with explanations that included diagrams and mathematical proofs that were followed by worked examples. Sets of practice exercises were provided beginning with purely procedural problems followed by application to real-world problems using word problems.

Toya Frank: My former district stopped using this text just as I arrived. The parents did not like it.

The third text I reviewed was Michel Serra's *Discovering Geometry* (2008). The text's pedagogical approach was Discovery Learning. In this textbook, as with the traditional textbook, topics were introduced with diagrams and text explanations for proofs. The explanations and proofs were followed by worked examples of problems. Exercise sets were then presented, in the same fashion as the traditional textbook, starting with abstract procedural problems that were followed by application to the real world with word problems. In a departure from the traditional text, there was an open inquiry project at the end of each unit that allows for further exploration of the topic.

System of Categories.

Two systems of categories were developed for this study that allowed the elements to be quantified by both type (e.g. nonverbal or textual) and purpose or use of the element (e.g. information or exercises).

The first categorization scheme developed was the system of types of nonverbal elements. Drawing from both the analysis of nonverbal elements in geometry textbooks by Gunzel and Binterova (2016) and Mayer, Sims and Tajika's (1995) categories of text and imagery resulted in an initial list of six items: Drawings, Geometric diagrams, Photographs, Tabular charts, and Text. After coding the first few pages of the *McDougal Littell Geometry* textbook I added the categories of Symbols/Icons and Equations/Math sequences to better reflect what was in the text. The final coding scheme was applied across all three textbooks. The types of elements were defined in the following ways:

- Nonverbal elements
    - Images
        - Drawings/Painting: An illustration that represents a real-world object.

        - Geometric diagram: An abstract line drawing that represents a geometric form that may or may not include dimensions
        - Photo: A photograph of a real-world object.
    - Nonverbal Other
        - Symbols/Icons: Any abstract graph image that was not necessary to the understanding of the materials.
        - Equation/Math Sequence: Equations or mathematic expressions. May contain text but only to identify variables.
- Text: Words in sentences used to communicate directly with the learner.

Sub coding of categories of usage were initially drawn from the Gunzel and Binderova categorization of nonverbal elements (2016) exercise, explanation, and inquiry and the Mayer, Sims, and Tajika (1995), resulting in the following categories: Exercise, Explanation, Inquiry, and Irrelevant/Nonmathematical. I added the category of Title to capture all of the text on the pages. The categories of usage were defined in the following ways:

- Exercise: Problems posed designed to provide sequenced practice after a topic has been introduced or explained.
- Explanation: Proofs, vocabulary, and descriptions of mathematical concepts.
- Inquiry: Materials designed to prompt further autonomous exploration by the learner.
- Irrelevant/Nonmathematical: Elements that do not explain or support exercises or inquiry.
- Title: Text that was not necessary for understanding the material in context.

The following additional protocols were developed and followed during coding of the materials:

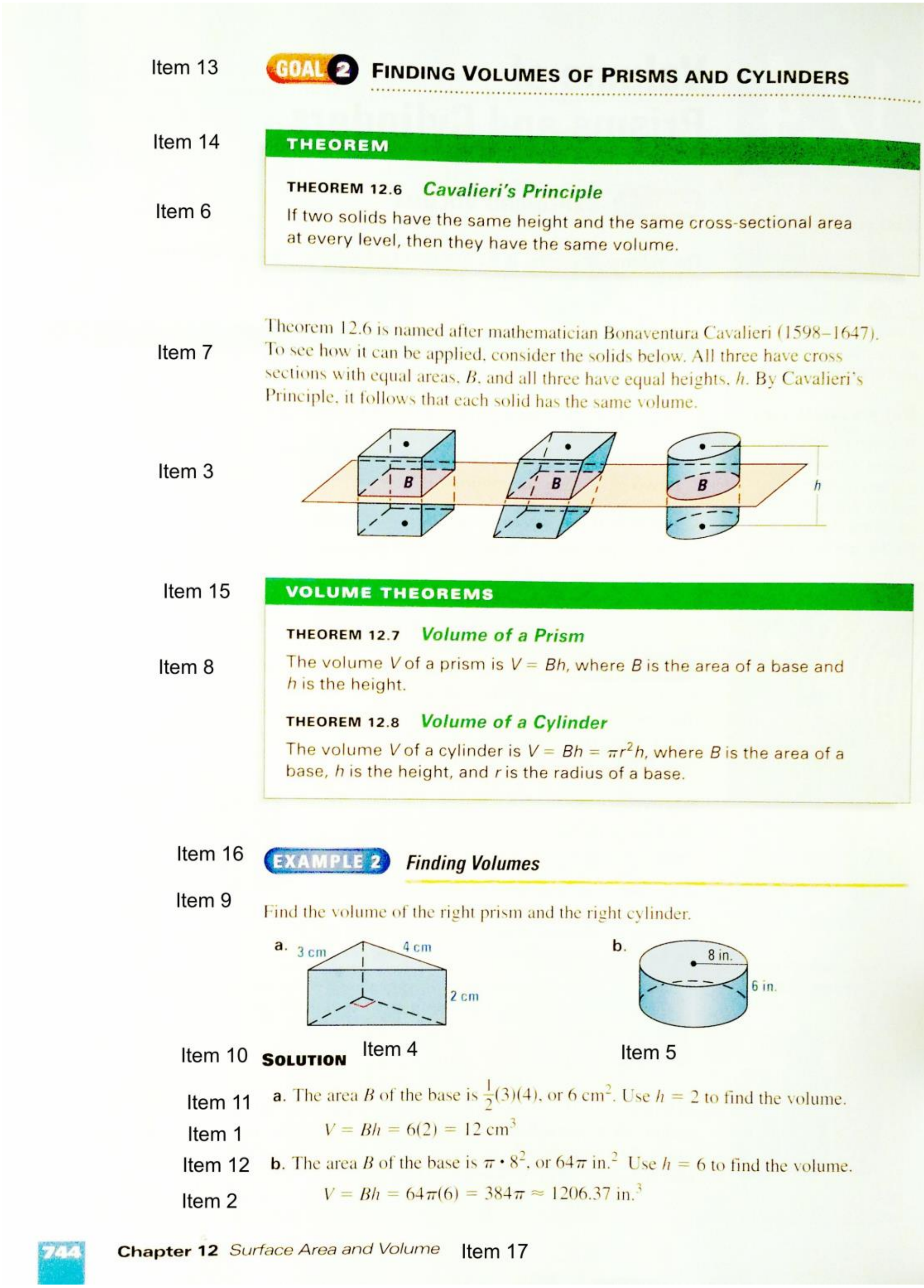
- Page numbers were not coded.
- Numbering of exercises was ignored unless it was visually unified with a text block or illustration.
- Graphic elements such as outlines or color blocks were ignored

Collection of Data and Calculation of Area by Category

Many comparative analysis classification and evaluation of nonverbal elements use frequency of occurrence as a measure (Gunzel & Binterova, 2016; Mayer, Sims &Tajika, 1995). This was problematic for this study because of the differences in the presentation and production methods of the textbooks. Consequently this study was designed to compare the proportion of each element analyzed.

Following the methods used by Mayer, Sims and Tajika (1995) I measured in centimeters the width and height of each element and then calculated the total area for the element. I captured the measurement and the categorization using survey forms I created in Google Forms for each textbook. The resulting databases (Figure 3) allowed me to sort the final results so that total area could be calculated and compiled by both category of item and usage in the textbook

Figure 2



Item 13 GOAL 2 FINDING VOLUMES OF PRISMS AND CYLINDERS

Item 14 THEOREM

Item 6 THEOREM 12.6 *Cavalieri's Principle*

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Item 7 Theorem 12.6 is named after mathematician Bonaventura Cavalieri (1598–1647). To see how it can be applied, consider the solids below. All three have cross sections with equal areas, *B*, and all three have equal heights, *h*. By Cavalieri's Principle, it follows that each solid has the same volume.

Item 3

Item 15 VOLUME THEOREMS

Item 8 THEOREM 12.7 *Volume of a Prism*

The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height.

THEOREM 12.8 *Volume of a Cylinder*

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

Item 16 EXAMPLE 2 *Finding Volumes*

Item 9 Find the volume of the right prism and the right cylinder.

Item 10 SOLUTION

Item 11 a. The area B of the base is $\frac{1}{2}(3)(4)$, or 6 cm^2. Use $h = 2$ to find the volume.

Item 1 $V = Bh = 6(2) = 12 \text{ cm}^3$

Item 12 b. The area B of the base is $\pi \cdot 8^2$, or 64π in.^{2} Use $h = 6$ to find the volume.

Item 2 $V = Bh = 64\pi(6) = 384\pi \approx 1206.37 \text{ in.}^3$

744 Chapter 12 *Surface Area and Volume* Item 17

Figure 2. A sample page from the *McDougal Littell Geometry* textbook with coded items listed in Figure 3 identified.

Figure 3

Item	Width cm	Height cm	Type	Context	Area
1	3.8	0.3	Equation, mathematic sentence	Explanation	1.14
2	6.5	0.4	Equation, mathematic sentence	Explanation	2.6
3	11.6	2.3	Geometric diagram	Explanation	26.68
4	3.8	1.9	Geometric diagram	Explanation	7.22
5	2.8	1.8	Geometric diagram	Explanation	5.04
6	11.5	1.4	Text	Explanation	16.1
7	12.4	1.6	Text	Explanation	19.84
8	11.6	3.2	Text	Explanation	37.12
9	8.6	0.3	Text	Explanation	2.58
10	1.7	0.3	Text	Explanation	0.51
11	12	0.7	Text	Explanation	8.4
12	12	0.4	Text	Explanation	4.8
13	11.8	0.4	Text	Title	4.72
14	2	0.4	Text	Title	0.8
15	4.2	0.4	Text	Title	1.68
16	5.5	0.4	Text	Title	2.2
17	6.5	0.3	Text	Title	1.95

Figure 3. Database of data collected from the textbook page displayed in Figure 2.

Because of the difference in scale between the three textbooks, comparative analysis based on raw data (cm^2) was meaningless (Appendix A). Unitization of the data by percentage of each type and usage allowed me to compare each across all three textbooks.

Results and Discussion (Table 1)

Representation by Usage

The differences in the pedagogical approaches of the textbooks may be evidenced by the overall proportion of usage (Table 1). The New Math textbook was designed to lead the student into discovering mathematical concepts so the text was designed to invite the student to consider multiple approaches to the topic. Consequently explanations (74.32%) used almost three times the area used for exercises (25.44%). The remaining area (.23%) was used for titles.

The Traditional textbook presented the topic didactically and followed with multiple worked examples and several pages of exercises. This resulted in a much greater proportion (57.98%) of the available area used for exercises rather than for explanations (29.48%). The

Toya Frank: wow. surprised, but not really.

Toya Frank: This is still lower than what I anticipated.

remaining area was used for inqu[redacted]1.96%), irrelevant/nonmathematical elements (5.75%) and titles (4.84).

It was not a surprise that the Discovery Learning textbook dedicated a significant portion of space to inquiry (2[redacted]%), resulting in a lesser proportion of area used for exercises (38.50%) than the Traditional textbook, and a lesser proportion of area used for explanations (32.00%) than the New Math textbook.

Types of Elements

Exercises. The proportion of textual space (81.92%) used by the New Math textbook for exercises was more than four times the space used by nonverbal elements (18.08%). The small percentage of nonverbal elements was a reflection of quantity and not size. While an analysis of the proportionate size of visual elements to the page size was beyond the scope of this study, a review of the unit reveals relatively large clear drawings when they are present. The small percentage of visual elements may also be a function of the pedagogical approach that encourages students to explore as they solve problems.

The split between nonverbal and textual elements of the Traditional (nonverbal: 57.05%, text: 42.95%) and Discovery Learning (nonverbal: 52.78%, text: 47.22%) textbooks are within a only few percentage points of each other. The difference in pedagogical approaches does not seem to have an impact on the division of space between nonverbal and textual elements in the exercises in these textbooks. One possible explanation for the similarity was the small four year gap in publication dates between the textbooks. Their similar use of space may have been a reflection of prevailing textbook design styles.

Explanation. The most surprising result from the study was the similarity in the proportion of space used for nonverbal and textual elements across all three textbooks.

Nonverbal vs. text results for the New Math, Traditional, and Discovery textbooks were 37.21% vs. 62%, 33.26% vs. 66.74%, and 37.61% vs. 62.39% respectively. Perhaps it was Euclid's influence. A breakdown of the nonverbal elements shows a vastly different split between images (photographs, drawings, diagrams) and other nonverbal elements (equations, symbols and icons). between the New Math (33.23% vs. 3.98%), the Traditional (18% vs. 14.9%), and the Discovery Learning (34.87% vs. 2.74%) textbooks.

Inquiry. Only the Traditional and Discovery Learning textbooks included inquiry elements. The Traditional textbook inquiry elements were all textual [illegible]0%). For the Discovery Learning textbook, the proportion used for nonverbal (38.81%) and textual (61.19%) elements were similar to the proportions that appeared in the explanation usage of all three textbooks. The similarity of the results to the proportion of nonverbal elements to text suggests that the same underlying forces may be influencing the results.

Toya Frank: Was there an expectation that the student would create the figure?

Irrelevant/nonmathematical, and title elements. While there were large differences in the percentage of space allocated to nonverbal and textual elements within the remaining usage types, the overall percentage of use was so low that it was reasonable to assume that the remaining elements have little impact on the learner.

Table 1

Percentage of Use of Nonverbal and Textual Elements

Textbook (Pedagological Approach) Useage	Total % of Useage Images*	Nonverbal Other**	Total Nonverbal	Text	Total Useage % of All Items
Geometry, Student's Text Part II Unit 14 (New Math)					
Exercise	16.14%	1.94%	18.08%	81.92%	25.44%
Explanation	33.23%	3.98%	37.21%	62.79%	74.32%
Inquiry	0.00%	0.00%	0.00%	0.00%	0.00%
Irrelevant/nonmathematical	0.00%	0.00%	0.00%	0.00%	0.00%
Title	0.00%	0.00%	0.00%	100.00%	0.23%
Total Type	28.81%	3.45%	32.26%	67.74%	
McDougal Littell Geometry (Traditional)					
Exercise	54.97%	2.08%	57.05%	42.95%	57.98%
Explanation	18.36%	14.90%	33.26%	66.74%	29.48%
Inquiry	0.00%	0.00%	0.00%	100.00%	1.96%
Irrelevant/nonmathematical	66.06%	0.00%	66.06%	33.94%	5.75%
Title	100.00%	0.00%	100.00%	0.00%	4.84%
Total Type	45.91%	5.60%	51.51%	48.49%	
Discovering Geometry (Discovery Learning)					
Exercise	31.18%	21.59%	52.78%	47.22%	38.50%
Explanation	34.87%	2.74%	37.61%	62.39%	32.00%
Inquiry	36.30%	2.51%	38.81%	61.19%	20.34%
Irrelevant/nonmathematical	85.15%	0.00%	85.15%	14.85%	6.79%
Title	0.31%	0.00%	0.31%	99.69%	2.37%
Total Type	36.33%	9.70%	46.03%	53.97%	

*Images includes drawings/paintings, geometric diagrams[illegible]ographs, and symbols or icons.

*Nonverbal Other includes equations/mathematic sentences and tabular elements

Toya Frank: So what is your interpretation of this data? I see the findings, but the discussion is not as evident in this section.

Limitations and Implications

The final coding schema allowed for an in-depth analysis of textbooks that spanned both multiple decades and differing pedagogical approaches. While the coding schema was applicable across the three textbooks represented, reliability should be confirmed with multiple coders. This would allow for refinement of the definitions for each usage type and nonverbal element.

As with all research, more possible questions are raised than answered by the results. Looking for differences in the textbooks that represented different approaches resulted in a surprising amount of similarities between them. Replication of the analysis with three different representative textbooks would support or refute the results of this study leading to greater understanding. A fine-grained analysis by type of image may have provided a clearer understanding of the results of this study, possibly revealing underlying causes similarities in spite of the differences in pedagogical approaches and decades of elapsed time between publication of the oldest text and the most recent.

With the rise of digital technologies the preeminence of the textbook in mathematics classrooms is fading making this study possibly a historical analysis. In spite of the impending obsolescence of physical textbooks the ability to do a successful analysis of such varying textbooks with the developed coding schema suggests that the method could be applied to other curriculum materials, including technology delivered materials. The categories would need to be expanded to include video, animation, and auditory elements.

Textbooks provide evidence for how topics may have been taught. The quantity and use of text and nonverbal elements at the very least may give us insight into the curricular intent of the authors. While the differences in approaches are made explicit in this study it is useful to keep in mind that in spite of the differences, curriculum developers are reaching for the same goal of developing successful math students and drawing from the same ancient traditions.

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Appendix A

Textbook (Pedagogical Approach) Useage	Raw Data (cm²): Nonverbal Images: Drawing/ Painting	Geometric diagram	Photo	Symbols/ Icons	Total Images	Nonverbal Other: Equation/ Math Sentence	Tabular	Total nonverbal	Text	Total Useage	Total % of Useage: Images	Total Nonverbal	Text	Total Useage % of All Items
Geometry, Student's Text Part II Unit 14 (New Math)														
Exercise		212.53			212.53	25.55		238.08	1078.72	1,316.80	16.14%	18.08%	81.92%	25.44%
Explanation		1278.26			1278.26	153.14		1,431.40	2414.97	3,846.37	33.23%	37.21%	62.79%	74.32%
Inquiry					0			0.00		0.00	0.00%	0.00%	0.00%	0.00%
Irrelevant/nonmathematical					0			0.00		0.00	0.00%	0.00%	0.00%	0.00%
Title					0			0.00	12.11	12.11	0.00%	0.00%	100.00%	0.23%
Total Type	0	1490.79	0	0	1490.79	178.69	0	1,669.48	3505.8	5,175.28	28.81%	32.26%	67.74%	
McDougal Littell Geometry (Traditional)														
Exercise	128.7	781.53	14.4	28.54	953.17	13.72	22.4	989.29	744.74	1,734.03	54.97%	57.05%	42.95%	57.98%
Explanation	30.89	127.96		3	161.85	131.37		293.22	588.33	881.55	18.36%	33.26%	66.74%	29.48%
Inquiry					0			0.00	58.66	58.66	0.00%	0.00%	100.00%	1.96%
Irrelevant/nonmathematical	1.55		98.85	13.15	113.55			113.55	58.34	171.89	66.06%	66.06%	33.94%	5.75%
Title				144.61	144.61			144.61		144.61	100.00%	100.00%	0.00%	4.84%
Total Type	161.14	909.49	113.25	189.3	1373.18	145.09	22.4	1,540.67	1450.07	2990.74	45.91%	51.51%	48.49%	
Discovering Geometry (Discovery Learning)														
Exercise	26.85	235.67	149.44	0.25	412.21		285.46	697.67	624.27	1,321.94	31.18%	52.78%	47.22%	38.50%
Explanation	41.02	197.04	145.17		383.23	30.1		413.33	685.61	1,098.94	34.87%	37.61%	62.39%	32.00%
Inquiry	14	162.02	77.57		253.59		17.55	271.14	427.49	698.63	36.30%	38.81%	61.19%	20.34%
Irrelevant/nonmathematical	2.5		194.5	1.45	198.45			198.45	34.6	233.05	85.15%	85.15%	14.85%	6.79%
Title				0.25	0.25			0.25	81.2	81.45	0.31%	0.31%	99.69%	2.37%
Total Type	84.37	594.73	566.68	1.95	1247.73	30.1	303.01	1,580.84	1853.17	3434.01	36.33%	46.03%	53.97%	