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Scope and Sequence

The professional development scope and sequence that will be discussed in this paper is a part of the larger professional development project: IMMERSION (Integrating Mathematical Modeling Experiential learning and Research through a Sustainable Infrastructure and an Online Network for teachers in the elementary grades), a NSF funded targeted professional development project focused on K-12 STEM teacher preparation. I will focus on the processes of two teachers who are part of the year long PD sequence that began with a summer institute that introduced Math Modeling to the teachers. Using a modified lesson study format, the participants integrated math modeling into their classrooms throughout the following school year. A subset of teachers were identified cases and selected for qualitative study. The two teachers who are the subject of this paper are part of this subset. Data collection included researcher memos, observation protocols, and interview transcriptions and data provided by the participants included written reflections and student work.

Content-focused coaching (West & Staub, 2003) is a professional development model that brings a coach and a teacher together in a sequence of dialogues to plan, teach, and reflect. The process includes diagnosing teachers' needs by identifying the gap between what teachers know and what they wish to accomplish. Teachers knowledge includes content knowledge, pedagogical knowledge, and underlying beliefs about learning.

Throughout the sequence I engaged the teachers in semi-structured and unstructured interviews that allowed them to bring to the foreground their knowledge of curriculum across multiple subjects and leverage the pedagogical content knowledge they have built over their

many years as teachers to design and deliver meaningful Math Modeling experiences for their students.

This paper encompasses the following sequence of interviews and observations:

- Pre-conference interviews were designed for lesson planning and reflection by the teacher of their perception of Math Modeling and the ways they would be able to implement MM in their classroom.
- Observation of the lessons allowed for collection of evidence of planning, pedagogy, mathematical content, and student learning.
- Post-conference interviews provided an opportunity for the teacher to reflect upon the lesson and the MM cycle.

Implementation

The participants in this sequence were Bonnie, a 3rd grade teacher with 23 years experience, and Lili, a 4th grade Japanese immersion teacher with 13 years experience. Both teachers brought deep knowledge of curriculum, pedagogy, and content knowledge to their teaching. Their involvement in the PD was evidence that both teachers were actively seeking new ways to engage their students in meaningful learning experiences.

In discussion posts made during the summer institute at the beginning of the process both teachers considered the impact of Mathematical Modeling on their practice and on their students. Lili wrote:

> "As I reflect closely on my own teaching, I begin to see that I need to learn to incorporate more mathematical modeling with open-endedness. My students get lots of opportunities to show various ways to solve problems, but I do not give enough math problems that could have multiple, possible answers. I'm hoping to learn more about mathematical

modeling so that my students will directly benefit from it to become better mathematicians." (discussion post, August 1, 2016)

Lili was looking to math modeling as a way for her students to explore math in an environment that allowed for multiple possible answers. Bonnie also was looking forward to giving her students more open-ended problems. In contrast to Lili's experience, Bonnie also considered what an open-ended *process* would mean for her students:

> "Since this is new to my students, I am wondering how they will respond to open-ended tasks. I suspect that I will need to do a lot of scaffolding and put parameters on the tasks. I can remove these as they progress in their willingness to try different ways to arrive at solutions. I am curious to observe their different responses and I am planning to keep notes on their ideas so that I can revisit them at a later date to see their growth." (discussion post, August 5, 2016)

Jennifer Suh: Kim, I loved how you went back to their posts from August to capture their aspiration to implement math modeling.

Jennifer Suh: I am thinking that it would be great to have the teachers revisit their posts and reflect on their journey. Ask them about critical incidents that mark a success and challenge in MM this year!

Pre-conference Interview

The pre-conference interviews for each of the teachers allowed for exploration of the range of learning possibilities using mathematical modeling in their classrooms. In their separate interviews both Bonnie and Lili expressed frustration in selecting a mathematical modeling problem that would be meaningful for the students and would challenge the students to apply skills that are part of the current year curriculum standards.

Lili. When I sat down with Lili she had already decided to create a project to refurbish the Japanese garden on the school grounds. She was struggling to find a way to incorporate meaningful mathematics, describing her conversation with a colleague about the project, "we started talking about adding some flowers that have some Japanese historical background to it, or

coming and measuring the perimeter of the garden. Or having our students look up scaling the garden or coming up with a map."

The garden contains a symbolic riverbed that is lined with dark and light gravel. Lili considered limiting their project in the garden to that area, "One idea that we had was that if we replace it with whiter stone or a prettier stone like how much pebbles would we need? ... if we are to cover this area we would need this much. Like how much, kilograms or we tried to use the metric system, so how many kilograms of pebbles would we need."

When I asked Lili about the standards for 4th grade math she responded, "what they do is the regular shaped quadrilaterals." Considering how this could be applied to the garden problem she commented, "I think the measurement definitely, but they also need to have some idea of the area ...I wonder if they would be able to figure out the estimated area? … we have to be kind of creative about it. If we draw a figure and somehow move the squares around it, what is the closest rectangular shape that we could use?"

After the interview, I received an email from Lili. It was clear that the pre-conference interview had prompted Lili to see geometric shapes in the environment in the way she would like her students to see them, " I noticed the meandering sidewalk which reminded me of the "river" in our garden. What I also noticed was the lines on the concrete … As the students know how to figure out the area of rectangles… I start to wonder if they will be able to estimate the area of the river by visualizing a series of adjacent rectangles in the river."

Jennifer Suh: Students discovering the best method for estimating the area of the irregular shape.

Bonnie. During the pre-conference interview Bonnie openly shared her frustration looking for a mathematical modeling problem that would be interesting to her students and mathematically relevant stating, "right now I'm kind of lost for ideas." She was approaching the

development by first considering possible activities such as a party for the students and then considering ways that mathematics could be integrated into the activity.

Considering Bonnie might have better success by starting with the mathematics instead of the activity, I asked her about the curriculum standards for 3rd grade mathematics and she responded, “Right now going into measurement. We have done fractions in the past, so we could get into that. Measurements.” Our discussion moved beyond mathematics into the other subjects that her students were studying. As Bonnie described her students’ science activities, the core of what she would build her mathematical modeling project on emerged, “Well our science is pretty much hands on, because they do things like investigate soil and things such as that ... I think maybe with school machines we could do something like that … they do have the difference in weight now that they could compare. And then basically they look at the time it takes for it to cross the finish line. But, you know, that's also very constructive event … they think it’s really a lot of fun when they go around and measure things, and compare them.”

Jennifer Suh: For Bonnie, she needed to see the mathematics more explicitly with the help of a coach.

The intersection of mathematics and science allowed Bonne to identify a mathematics modeling activity that she believed would interest her students and build their knowledge in both subjects. During the interview, once she had identified a viable project she immediately moved into planning mode, “maybe if they saw a short video clip of different things going at different speed, so like, Pinewood Derby type thing. And like, they could talk about well, uh, what do you notice about them, that type of thing. But, uh, one thing I learned from the last mathematical one, is you start about what kind of mathematical questions could you ask about this experience.”

Jennifer Suh: Questioning as a tool to get the mathematics out to the forefront.

Lesson Observations

Two observations were conducted for each of the teachers. One at the launch of each of their MM projects and one in the middle of the cycle as their students were preparing their posters for presentation of their project results.

Lili: During the observation of the launch of the project, Lili's class was already familiar with the garden, having walked the space previously. Lili reintroduced the garden with a photograph of the garden and opened conversation about what could be done to refurbish it. After the students gathered into groups to discuss possibilities, Lili called their attention to the board where she had attached poster paper to record their ideas. As she asked them questions about the garden project she elicited ideas from the students about area and measurement, asking them to recall their previous work with area. This was followed by instruction to the students to gather measuring and recording tools from the classroom (meter sticks, tape measures, a measuring wheel, and clipboards) and to work in groups in the garden to gather information. In the garden students were seen walking the space, drawing the general shape of the riverbed, and recording measurements. There were several animated discussions within the groups of the best way to capture data from the garden.

In a later observation the student groups were working in the classroom preparing poster presentations of their findings. Though each of the students' posters showed use of rectangular shapes to estimate the area, none of the layouts were the same and each group's' results were different. One group of students had also recorded their *miscalculations* and the process they had used to correct their approach to the problem.

Bonnie. Bonnie introduced the project by asking the students if they'd heard of the pinewood derby, the annual Boy Scout model car competition. Students responded by eagerly raising their hands and waving them to get attention. Bonnie played a video of several runs of

pinewood derby cars. Afterward she asked the students what they noticed. The students respond with comments about speed and time. She used these responses to lead the students into a discussion of how math can be used and what information could be captured. She recorded the student's responses on large sheets of paper that she had attached at the front of the room. At the end of the discussion she asked the students "How would you design a car and measure how fast it would go," then released them to create drawings of their ideas. Students were seen drawing both ramps and cars.

Bonnie's students were experimenting with their ramp designs and measuring travel distance during the second observed class session. Working in groups and using materials found in the room, they ran model cars made from K'NEX rods, connectors, and wheels down ramps made of cardboard. Bonnie asked the students how many centimeters  meter stick as a student laid several end-to-end along the runway her car had just traversed. The student quickly correctly calculated the total length of her car's distance traveled by multiplying the quantity of meter sticks and adding the measured distance from the end of the last meter stick to the front of her car. As the group worked, Bonnie asked them to consider the best way to record each of their runs.

Jennifer Suh: Kim, you mentioned how Bonnie was quite skillful in engaging her students in discussion. It would be interesting to pull out the questions she asked her students especially since she thought about questioning as a tool for getting the math out to the forefront.

Post-conference interviews

Lili: In the post-conference interview following the second observation Lili talked about her need to scaffold her students' understanding of area when it became apparent that the relationship between area and perimeter was not clear to them, "I had three seconds of silence standing in front of them. Oh, ok, you know, keeping the smile on my face, and then saying that's ok let's review … being actually the third grade math concept … I just assumed that the fourth graders … would actually be able to use that concept into the real life."

She described how she used several types of worksheets with her students to go the concept of area using dot patterns and grids to divide irregular shapes into the numbers, “I had to make sure that they understood that we are dealing with an irregular shaped area ... They had to literally count the squares within the grid to figure out the area.” She was comfortable allowing the students to struggle for a while as they made sense of the concept of area, “I want them to struggle and then I want them to realize that the way they were trying to figure out the area by just measuring the perimeter was not the way to go.”

Jennifer Suh: allowing for Productive struggle

I asked her about the intentional demonstration of miscalculation on one group’s poster. She shared that she uses students errors as learning moments by telling them, “you'll be making mistakes along the way. And those are the mistakes that I need to know, you need to know, they need to know. So that, those are the precious learning moments that I don't want them to get what the mistakes were, let alone, how they found it. So I told them, someone needs to make a note.”

Jennifer Suh: Embracing mistakes and learning opportunity

Discussion with Lili of her students’ productive struggle led to a reflection of her own struggles designing the project and how the process changed her own perceptions, “because we ourselves had a little productive struggle ... remember the day we came to visit you? … How clueless we were at the time … we still didn't have a concrete idea ... And then, you know, I felt really good after our meeting, that, um, maybe I can lead them to think that they can use the rectangular, you know, method … As we walked out of the building we saw rectangles on the pavement … I was screaming, I was screaming, look! And then they're everywhere.”

Jennifer Suh: I love that parallel reflection she made.

In spite of her initial frustrations designing the project she was eager to start her next MM project, “I can't wait to come up with another one. Um, it is true that I've tried different, you

know, um, workshops to always keep improving as a teacher ... Among all that, probably this really has fundamentally changed the way I look at math teaching al[redacted]her."

Jennifer Suh: THat is powerful! Do you think she felt like she had more ownership because she was the designer of this MM design

Bonnie. Bonnie had expressed concern that the project needed to be interesting to her students. In the interview following the second observation she felt that the students had been successfully engaged, "I think it went pretty well ... And that was the little cars that they made and the Pinewood Derby make and that association where they had that experie[redacted]a lot of them had it already. And also it's a topic that there's a lot of videos on, that are available on YouTube, and if I show plenty amount of those it kind of gets them thinking again." She was very happy about the students' excitement over the project, "Some of them were shouting at each other, we had to go through and say "you're right next to each other you don't have to shout". So, but you know, that's excitement and that's a good thing right now. It would be horrible to have a group that wasn't excited and try to do this, you know?"

Jennifer Suh: tapping into her students interests and relate-able

She felt that the charts that she had created with the students during the discussion following the pinewood derby videos had been effective, "I planned out the [redacted]rent charts ... that would be the questioning phase … first they have to notice things and then they have to ask their questions. And that went pretty well."

Jennifer Suh: Tell me more kim. What was these charts?

When I asked Bonnie about how the students were using math in the project she explained that the besides the process that I had observed, she had been surprised that the students had used multiple methods to measure the distance their cars had traveled, "mathematically, there was someone mentioning about the distance. You could use a meter stick to measure the distance of how far they went. Um, it was interesting to see this one group of boys had made this really elaborate track. Start it there, went down, went all the way up to about the second group of tables. Um, they had a measurement system, but they were only measuring from one little area

… they used the meter stick but they didn't use the centimeters, instead they put washer on it to see how far it went." She was also surprised at the insight of a girl student who is an English language learner, "she's pretty much, uh, one without very much experience, and she's the one that brought in … the idea "angles". And no one else had mentioned angles. And she said that angle the car started from."

Bonnie considered her own struggles working with the openness of MM as we were discussing her students' response to open-ended problems, "Because if it's, you know, if things are open ended there's a lot of insecurity rather than if things are defined and they know exactly what to do. Same as teachers you know – if you have to design an open-ende[redacted] it's like "well what do I do" .... And I'm still working my way through it, so it's still very open-ended. It's still, well I still don't know how it's going to turn out."

Jennifer Suh: This is key because I think this is the challenge that most of our IMMERSION teachers have and some try modeling and don't get to a point where they get beyond these insecurities.

Bonnie's students had not completed the project at this point so we discussed what her plans were as she completed the MM cycle. She expressed concern for the quantity of mathematics that the students had been able to pull from the project so far. She was making plans to address this, "I'm going to emphasize finding the math, where's the math? … We're going to look for elements that could measure or some type of math connection with it. Yeah, we're going to see where that goes."

Evidence of Progress

Change in Lili's and Bonnie's teaching can only be judged through their reflections since observations of their teaching before this PD did not take place. Though both teachers had been through the Mathematics Modeling cycle before during the design stage they still expressed frustration developing a project that would both engage their students and elicit mathematically appropriate inquiry by their students. The unstructured interviews allowed them to settle on a

Jennifer Suh: Abiility to see "MM eyes"

Jennifer Suh: Ability to connect and see the continuity of learning from math and science!!!

plan to design projects that were engaging and mathematically meaningful to their students. It is clear from Lili's reflections that this experience has influenced her ability to see [redacted] in the environment. Bonnie's MM ideas began to fl[redacted] once she considered the curriculum standards for science. Neither class has completed their projects as this writing so it is still an open question how much the teachers' perspectives have changed and what the long-term impact may be on their teaching.

Both Bonnie and Lili mentioned the possible impact this experience may have for their students beyond the classroom. Lili commented that, "I can send a letter home saying that the students have learned how to, you know, uh, measure the rough figure. Wouldn't that be wonderful if you let your child figure out how many bags of mulch that your family needs to cover your, you know, yard ...So that the whole family would be involved in math modeling process." Bonnie too wrote about the possible impact that MM might have on students' families, "As engagement increases, students are more likely to talk about their experiences at home, increasing parental buy-in."

Reflection

Both teachers' described our interaction as helpful to their design process. My perspective on the nature and impact of the interviews was that I functioned as a sounding board to reflect back to both teachers their clear understanding of the content, of the curriculum, and of their students. When asked what supported her through the MM process Bonnie wrote, "Kim helped me to think further with her question. "What are they already studying?" This helped me focus on measurement, our current topic, and the cars and inclined planes- part of our science curriculum." Lili said, "So if it wasn't for that meeting, um, I don't think I would have, you know, figured out that this is something that my fourth graders could do."

Both Lili and Bonnie demonstrated in many ways the depth of their mathematical knowledge for teaching (Hipp, Sleep, Lewis, & Ball, 2007): their specialized professional understanding of how knowledge of mathematics is applied in the classroom. When Lili's students demonstrated confusion about area calculations, she was able to pull resources that would allow the students to connect with their earlier learning experiences. Bonnie's familiarity with both the science and mathematics standards allowed her to easily envision a project that would encompass both.

Interviewing and observing these very accomplished teachers allowed me to share in their learning experience. Both teachers have a deep understanding of curriculum and a clear picture of the expectations and skills of their students. Their openness in sharing their frustrations and struggles adapting to a new teaching process helped me to see as a facilitator how important it is to be mindful of their needs as learners to discover for themselves how to use the MM process in ways that were appropriate for their students. Allowing both teachers the time and space to explore possibilities was essential to their learning process (Loucks-Horsley, Stiles, Mundry, & Love, 2010) As they described the difficulty of determining the correct amount of scaffolding for their students, I became keenly aware of my own struggles determining how much to scaffold Bonnie's and Lili's learning processes. Ultimately it was clear to me that in the same way that they had helped their students connect their previous knowledge to their current activities in the MM project I had been able to assist their project development by helping them to build on their previous knowledge and years of experience to create lessons in the less familiar mathematical modeling form.

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