857 Research Synthesis and Annotated Bibliography

Technology and mathematics have always existed in a symbiotic relationship, circled each other, the demands of one pushing the development and novel implementation of the other. While this relationship is accepted in science and engineering, integration of digital technology into the mathematics classroom has not been as accepted as one would expect

In 1985 graphing calculators were introduced into classrooms with great fear that students would lose their ability to calculate and graph by hand (Tirosh & Graeber, 2003). The fears were unfounded and later research found that access to calculators had a positive effect on student problem solving when the curriculum was designed to leverage teacher pedagogical and content knowledge to incorporate the affordances and constraints of the tool (Kastberg & Leatham, 2005).

In the intervening years, the importance of the effective use of technology in the mathematics classroom has been firmly established ("Principles to Action", 2014; NCTM, 2011). The National Council of Teachers of Mathematics position on the role of technology in the teaching and learning of mathematics posted to the NCTM website states:

> "It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students" (NCTM, 2011).

In Virginia, the NCTM position is embodied in technology standards for students and teachers. The standards of student skills have been defined by the Contents of the Virginia

Jennifer Suh: This expectation aligns with CSTA's new computational thinking push.

Jennifer Suh: That is interesting!!!

Computer Technology Standards of Learning (Board of Education, 2013). Students are expected to be able to gather data, create and use models and simulations, and apply knowledge skills to generate innovative ideas and solutions. For teachers, the Virginia Administrative Code (Technology Standards for Instructional Personnel of 1998) addresses the skills and knowledge that teachers should have to be able to effectively use technology in the classroom and in professional practice. The standards for teachers include the use of technology for data collection, information management problem solving, and planning and implementation of lessons that integrate technology to meet diverse learners' needs.

In spite of the near universal calls for the integration of technology into classrooms and overall improved access to computer technology in schools, teacher surveys show that use and integration of computer technology in classroom activities is actually in decline (Wachira & Keengwe, 2011). The challenges that teachers face when trying to integrate technology into their classrooms include continuing lack of resources for some, teacher beliefs and attitudes about using technology in teaching, and lack of training (Kastberg & Leatham, 2005; Koehler & Mishra, 2009).

Teachers daily navigate a complex environment driven from above by mandates and standards in their classrooms. Ultimately it is their knowledge and beliefs about technology that determines how or if they will use technology tools in their teaching practice.

Teacher Knowledge

Technology has potential to enable dynamic modeling leading to more constructive and experimental forms of mathematics and allow learners to explore and investigate context to deepen understanding and provide insight into mathematical concepts (Van Wert, 1998). In order to harness the affordances that technology may provide, teachers must possess a particular set of knowledge and skills.

To assess what constitutes teacher knowledge in relation to technology use in the classroom, Mishra and Koheler (2006) incorporated content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK) into the technological pedagogical content knowledge (TPCK) conceptual framework. TPCK seeks to provide a means to describe the ways in which the three forms of knowledge and the relationships among and between them are brought to bear when teachers integrate technology into their classroom environments. Interaction among these forms of knowledge is evident when teachers successfully synthesize these competencies to implement technology based learning environments (Chen, Huang, & Chang, 2013, Drijvers, 2013).

Jennifer Suh: Very relevant literature to include to frame your work when working with Technology and teacher education. Great!

Schulman (1986) originally introduced the conceptual framework of pedagogical content knowledge (PCK) as a way to discuss and define the ways in which content knowledge (CK) and pedagogical knowledge (PK) are intertwined and interdependent in effective teaching practices. Using Schulman's original work, Misra and Koheler (2006) expanded the concept to include technological knowledge. They argue that the development of the model was necessary in part because the original PCK model could not foresee the impact that technology would have on classrooms and teacher practice. More than a source of curriculum content, technology in the classroom fundamentally changes the relationship between CK and PK. It is at the boundaries of the three forms of knowledge that the greatest complexity occurs. The TPCK framework allows not only for a way to operationalize the concepts, but also an acknowledgement of the complexities challenges that are created when teaching in technologically rich environments (Mishra & Koheler, 2006).

Jennifer Suh: Important to consider as a designers...what are the specific challenges and complexity presented in math teaching with tech?

Jennifer Suh: Definitely a clear affordance!

Jennifer Suh: Key to framing your work... instrumental genesis...

When TPCK integration is manifest in a classroom, technology can be used to aid in the development of students' representational fluency. Technology vastly increases the availability of multiple representations in type and variety that can be adapted to the teachers' and the students' needs over time (Koehler & Mishra, 2009). TPCK allows the teacher to configure the interaction in a way that students can explore and reflect on the connections between actions and changes in the representation. An environment that requires students to move among multiple representations of the same concept is more likely to provide cognitive support for the development of representational fluency (Zbiek, Heid, & Blume 2007). In order for this to occur the learning environment must provide "rich sets of actions that will expose underlying invariances and thus enable the student to heave a flexible and enduring web of mathematical meaning" (Kaput, 1989, p. 180).

As teachers engage their knowledge and skill to implement technological tools into the classroom, the tool can become symbol for, and a repository of, cumulative and shared knowledge. Instrumental genesis is the ability to use technology as a tool to build and support mental schema. When instrumental genesis is in play the tool allows for not only the completion of a task, but also a deeper conceptual understanding (Drijvers, 2013). While deepening conceptual understanding may be teacher's ultimate goal, technology also allows for exploration of content in ways that can exceed both the teacher's and the student's skills and understanding (Hegedus & Moreno-Armella, 2011). This very provocative circumstance bears deeper study but is beyond the scope of this paper.

In light of the nature of TPCK it comes as no surprise that for a professional development program that involves technological tools to have lasting change practice it must do more than simply present a new tech tool to the teacher. Research indicates that when new technology skills were introduced and teachers were also given the opportunity to explore ways to incorporate the tool into their mathematics instruction, both teacher content knowledge and technological skills increased. The greater the increase in technology skills, the more likely teachers were to use technology in their classrooms (Hartsell, Herron, Fang, & Rathod, 2009).

Teacher Beliefs

While knowledge and skills are essential to the successful implementation of technological tools in the classroom, implementation won't happen at all unless a teacher believes in th[illegible]rpose of the tool, and the tool's ability to support teaching and student learning (Kastberg & Leatham, 2005).

Jennifer Suh: Critical area to attend to with teachers in PD.

Beliefs differ from knowledge in that beliefs are felt to be true and knowledge is viewed as "belief with certainty" (Clement as cited in Philipp, 2007, p. 266) When considering teachers' beliefs about technology integration is is important to keep in mind that in the classroom beliefs are bound to the activities and cultural contexts in which they are found (Brown as cited in Philipp, 2007).

Though fear of technology has been identified as a possible reason for low technology integration (Mishra & Koehler, 2006), research suggests that teachers' underlying beliefs and confidence in their ability to use technology may be more salient drivers. Wu, Chang, and Guo (2008) found that ease-of-use was n[illegible]predictor of teachers' use of technology in their classrooms. Instead, perceptions of usefulness for improvement of teachers' performance and student learning, and strong computer self-efficacy were found to significantly increase the probability of technology integration. Perceived usefulness to improve teacher performance and student learning and strong computer self-efficacy (confidence in one's ability to use computer

Jennifer Suh: Usefulness and efficacy– as stronger predictors...

skills to execute a task) were found to predict the likelihood that science teachers would use IT in their classrooms (Wu, Chang, & Guo, 2008)

Earlier research also indicated that when the opportunity to develop lesson plans was combined with training on a technology positively impacted teachers' attitudes toward the use of technology in their classrooms and had a positive effect on teachers' acceptance of technological advances in their classrooms (Hardy, 2004 Trosh & Graeber, 2003). And though attitudes are more transitory than beliefs (Philipp, 2007), there is no doubt that they contribute to a teacher's overall perception of the value of technological tools.

Conclusion.

The perception of the usefulness of a technology tool is bound by a teacher's ability to integrate the tool as only one of many means to present concepts and applications. The interaction between technology and content both affords and constrains the types of ideas that can be taught (Koehler & Mishra, 2009). An understanding of these affordances and constraints can only empower a teacher who has well-developed content knowledge and pedagogical knowledge.

While high degrees of technological pedagogical content knowledge are necessary for a teacher to successfully implement the use of technological tools in their classroom, implementation itself appears to be heavily dependent upon the teacher's belief systems.

Jennifer Suh: So knowledge about TCPK;persuasion of usefulness; co-designing lessons integrating tech have to be attended to early and in tandem in PD with Tech....

Annotated Bibliography

Assiri, M.A (2003). *Exploring the attitudes and practices of northwest Arkansas high school mathematics teachers regarding technology*. Ph.D. thesis, University of Arkansas.

"Current use of computers and the internet … is primarily driven by the teachers' experience, interest, and internal drive to use the technology in math education."

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Board of Education (2013). *Computer technology standards of learning for Virginia's public schools grades 9-12*. Commonwealth of Virginia.

Bos, B. (2008). Mathematical and cognitive fidelity, technology impacting mathematical achievement. In K. McFerrin, R. Weber, R. Carlsen & D. Willis (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2008* (pp. 4404-4406). Chesapeake, VA: Association for the Advancement of Computing in Education (AACE).

Buchberger, B. (n.d.). *Special didactic activities/the white-box/black-box principle for using symbolic computation systems in math education*. Retrieved from http://www.risc.jku.at/people/buchberger/white_box.html

Introduction of the White-Box/Black-Box Principle. When a student in the white-box phase, when the topic is new to a student, systems should not be used. When a student is in the black-box phase, complete mastery of a new topic, it is essential for students to use systems. The phases are recursive.

Bourassa, M. (2014). Technology corner - Desmos activities. *Gazette - Ontario Association for Mathematics, 52*(4), 8-10. Retrieved from https://search-proquest-com.mutex.gmu.edu/docview/1563633425?accountid=14541

Chen, C.H., Huang, C.K. & Chang, C.Y. (2013). Teacher Professional Learning in Applying the Geometer's Sketchpad to Mathematics Instruction. In J. Herrington, A. Couros & V. Irvine (Eds.), *Proceedings of EdMedia: World Conference on Educational Media and Technology 2013* (pp. 1732-1738). Association for the Advancement of Computing in Education (AACE).

PLC created for adoption of Geometers' Sketchpad in the classroom. Qualitative data, case study. Found that though case study participants demonstrated skill with technology did not transfer to the new skills. Middle school teachers (4) iterative process of lesson planning and worksheet development. Complex interaction among PK, CK and TK were noted when teachers engaged in a PLC.

Darling-Hammond, L., Goldman, S., and Zielezinski (September 10, 2014). *Using technology to support at-risk students' learning. Alliance for Excellent Education and Stanford Center for Opportunity Policy in Education*. Retrieved from https://edpolicy.stanford.edu/sites/default/files/scope-pub-using-technology-report.pdf

Drijvers, P. (2013). Digital Technology in Mathematics Education: Why It Works (Or Doesn't). PNA, 8(1), 1-20.

Review of leading research studies on tech in math ed. Handheld graphing calculators (2010). Five modes of tool use, computation, transformation, data collection and analysis, visualisation, and checking. Teacher was crucial in establishing and reinforcing modes of tool use, demonstrating the ability to synthesize understanding of the use of tech tools (instrumental genesis) and knowledge.

Defines Instrumental Genesis framework as the instrumental approach to tool use, or the ability to use technology as a tool to build and support mental schema. So, a tool allows for the completion of a task and contributes to conceptual understanding.

Gerny, M. and Alpers, B. (2004). Formula 1 - A mathematical microworld with CAS: Analysis of learning opportunities and experiences with students. *International Journal of Computers for Mathematical Learning, 9,* 25-57.

Gogus, A. (2012). Action research on learning. In Steel, N. M. (Ed), *Encyclopedia of the sciences of learning*. (pp. 69-72). New York: Springer

Gordon, M. (2016). *Enabling students in mathematics: A three-dimensional perspective for teaching mathematics in grades 6-12*. Switzerland: Springer International Publishing

Habits of mind - the heart of the mathematics curriculum: Some instances.

Technology allows a student to see the mathematical relationships present in algorithms.

Greenwold S. J. and Thomley, J. E. (2013). Proceedings of the Twenty-fourth Annual International Conference on Technology in Collegiate Mathematics. *Using the history of mathematics technology to enrich the classroom learning experience*. New York: Pearson.

History of tech use in education going back to antiquity. By 2004 Mathematical Assn of America recommended students should understand impact of computer tech on course content. Impact specific to topics is discussed.

Hardy, M. (2004). The Technology In Mathematics Education (TIME) Project: Preparing Secondary Mathematics Teachers To Teach With Technology. In L. Cantoni & C. McLoughlin (Eds.), *Proceedings of EdMedia: World Conference on Educational Media and Technology 2004* (pp. 3322-3327). Waynesville, NC: Association for the Advancement of Computing in Education (AACE).

Cites sources highlighting insufficiency in teacher knowledge of tech resources and methods from the late 1990s. Initial course covered tech tools and asked participants to evaluate tools and develop lesson plans and activities with tech. Follow-up meetings to support participants' efforts: discussion, additional methods, and resources.

Questionnaire covered frequency, perceptions, and preparedness to teach with tech. Pre/post tests were compared. Open ended question asked participants to outline a lesson that included tech. TIME project significantly impacted participants' perceptions of ability to teach with tech and knowledge of technological resources. Small number of participants. PD positively impacted participants' perceptions indicated by change in survey responses.

Hartsell, T., Herron, S., Fang., and Rathod, A. (2009). Effectiveness of professional development in teaching mathematics and technology applications. *Journal of Educational Technology Development and Exchange*, *2*(1), 53-64.

Research indicated that all participants agreed that they gained knowledge from a four week training on software, calculators and other tools available in classrooms resulting in effective integration. Important was use of samples that were relevant to concepts that would be taught in actual classrooms, increasing teachers' knowledge of solving equations. Teacher content knowledge increased. Confidence increased. Results indicated that teachers needed to be taught both technology skills and how to incorporate into math instruction. The more the teachers technology skills learned in the PD, more likely to use tech in their classrooms. Teachers need to learn both tech skills and now to incorporate into classrooms.

Hegedus, S.J. and Moreno-Armella (2011). The emergence of mathematical structures. *Educational studies in mathematics, (77)*2, 369-388.

Use of technology in the math classroom caused an epistemological rupture similar to the emergence of non-euclidudian geometry. Math structure is embodied in the technology, a cognitive artifact that allows us to exceed our biological frame.

Hill, H. C., Sleep, L., Lewis, J. M., and Ball, D. L. (2007). Assessing teachers' Mathematical Knowledge: What knowledge matters and what evidence counts? In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-155). Charlotte, NC: Information Age Publishers and National Council of Teachers of Mathematics.

Assessing teacher mathematical knowledge: General, in the classroom, professionally situated mathematical knowledge, mathematical knowledge for teaching

Kaput, J. (1989). Linking representations in the symbol systems of algebra. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 167-194). Reston, VA: National Council of Teachers of Mathematics.

Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan Publishing Company

Kastberg, S. and Leatham, K. (2005). Research on graphing calculators at the secondary level: Implications for mathematics teacher education. *Contemporary Issues in Technology and Teacher Education, 5*(1), 25-37.

Factors that impact graphing calculator use: access, place in the curriculum, connection with pedagogical practice. Recommends that teachers reflect on practice using video.

Koehler, M. J., and Mishra, P. (2009). What is technological pedagogical content knowledge? Contemporary *Issues in Technology & Teacher Education, 9*(1), 60–70.

Demonstration of need for content knowledge, ped knowledge, and tech knowledge to successfully integrate tech into teaching.

For teachers**:** Digital technologies are rapidly changing and can be used in different ways and how to use them can be opaque to the user. Teachers often have inadequate experience with using digital tech for teaching and learning. Tech must be consistent with existing pedagogical beliefs.

TPACK framework three components of teachers' knowledge - content, pedagogy, and tech.

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N. and Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, California: Corwin

Liang, S. (2016). Teaching the concept of limit by using conceptual conflict strategy and Desmos graphing calculator. *International Journal Of Research In Education And Science*, *2*(1), 35-48.

Discusses the importance of the understanding of limits to advance mathematical understanding. Possible lack of understanding in calculus teachers (content knowledge). Uses Desmos to demonstrate what happens in a function with infinite limits.

Description of Desmos (40). The calculator is not employed until students have tried to solve manually. Anticipates a conceptual conflict when the Desmos graph does not match the students' predictions. Ultimately hoping to cause student's conceptual change. Used as a tool to teach formal and rigorous mathematics. Considers tech an auxiliary to learning.

No discussion here about training to use Desmos.

Loucks-Horsley, S. Love, N, Stiles, K., Mundry, S., & Hewson, P.W. (2008). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin Press, Inc.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Defines TPCK conceptual framework Technological Pedagogical Content Knowledge: content, pedagogy, and technology. Discussions of integration in theoretical, pedagogical, and methodological. Understanding teachers' development toward rich uses of technology.

MacIsaac, D. (2016). Desmos: A free cross-platform calculating and graphing tool. *Physics Teacher*, *54*(8), 69. doi:10.1119/1.4965284

Description of Desmos and a recommended tutorial.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054. DOI: 10.1111/j.1467-9620.2006.00684.x

Seminal work (over 1k cited). Conceptual framework for Tech Ped Content Knowledge (TPCK). Integration of three components of learning environ: content, pedagogy, technology. Teachers may have fear of change or lack of time (1023). Need to

be able to adapt as technologies become obsolete. Understanding of the representation of concepts using technologies. Learning is situated, so tech is context bound (think about training to the context that the teacher will be in). “The ideal that the design of educational technology represented an authentic context for teachers to learn about educational technology.” (This may be justification for placing the PD online)

Learning process compelled each member to seriously study tech, ed, and where they meet. Learned about tech as they needed to in order to fulfill a feature of the course. TPCK framework (1039)

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age Publishers and National Council of Teachers of Mathematics.

A comparison of the constructs of knowledge, belief, and values. How they impacted by teaching experiences, and how they impact teaching practice.

Principles to actions: Ensuring mathematical success for all. (2014).. Reston, VA: National Council of Teachers of Mathematics,

Rivera, F. D. (2011). *Toward a more visually-oriented school mathematics curriculum*. New York: Springer Science+Business Media.

Mathematical visualization. Use for information Instrumental Genesis: how visual images and/or processes change as learners transition from one type of representation to another. Using a TI-89 lessons designed that as students learned the tool they were also developing a deep understanding of mathematical functions. Technological tools allow for the “progressive evolution of mathematical thought, from the concrete to the abstract or from material to theoretical knowledge.” (pg. 11). Knowledge becomes a tool mediated activity.

Schulman, L. S. (1986). Those understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Defining article for pedagogical content knowledge (PCK)

Staley, J.W. (2006). Examining electronic learning communities as a means for sustaining and supporting mathematics professional development. Ph.D. thesis, George Mason University.

Blended face-to-face and online activities were the preferred follow-up method of interaction. Face-to-face was preferred on a shorter (10-12 week) cycle (vs. quarterly). Teachers reported using technology more for personal use than professional. Asynchronous communication tools are beneficial. Practice and reflection contributed to understanding.

Steeds, A., & Basic Skills Agency, L. (. (England). (2001). *Adult numeracy core Ccrriculum*.

Strategies for increasing adult numeracy in the UK.

Technology Standards of Instructional Personnel, 8VAC20-25-30 (1998).

Technology standards for teachers state of Virginia

Trosh, D. and Graeber, A. O. (2003). Challenging and changing mathematics teaching classroom practices. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, and K.S. Leung (Eds.), *Second international handbook of mathematics education*. Dordrecht, Netherlands: Springer Science + Business Media.

Values and beliefs affect teacher acceptance of tech advances. Use of organization and pd to affect teacher change.

Van Weert, T. J. (1998). The impact of informatics on the teaching of mathematics. In D. Tinsley & D. Johnson (Eds.), *Information and communications Educational Technologies and Mathematics 39 technologies in school mathematics* (pp. 7–18). London, England: Chapman & Hall.

Enabling the teacher and learner to explore and investigate context to deepen understanding and provide insight into mathematical concepts.

Vause, L. P. (2009). *Content and context: Professional learning communities in mathematics* (Order No. NR60874). Available from Education Database; ProQuest Central; ProQuest Dissertations & Theses Global. (375441436). Retrieved from https://search-proquest-com.mutex.gmu.edu/docview/375441436?accountid=14541

VeraQuest, Inc. (January 2013) Teacher technology usage. *PBS LearningMedia*. Retrieved from http://www.edweek.org/media/teachertechusagesurveyresults.pdf

Survery conducted for PBS LearningMedia to understand the amount and type of technology in classrooms, how technology is used, teacher attitudes, and training needs.

Teacher beliefs that because kids are digital natives, classrooms should embrace 21st century skills.

Wu, W., Chang, H.P. and Guo, C.J. (2008). An empirical assessment of science teachers' intentions toward technology integration. *Journal of Computers in Mathematics and Science Teaching, 27*(4), 499-520.

Technology Acceptance Model (TAM). Study of teachers' intention to use technology, perception of usefulness, ease of use, actual use. Study added computer self-efficacy (CSE), and task-technology fit (TTF) constructs previously used to study technology acceptance in workers and consumers. Found that science teachers were likely to use IT because they believed tech sould improve their teaching performance and student learning. CSE is a motivator.

Zbiek, R. M., Heid, K. M., and Blume, G. W. (2007). Research on technology in mathematics education. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age Publishers and National Council of Teachers of Mathematics.

Divides mathematic activitiy into technical and conceptual.

Technical is mechanics/procedural technoloy allows for "off-loading" routine computations, efficiency. **Graphs** can be examined quickly identify common attributes alowing for connections. Compacted tech activity affords an opportunity for enriched conceptual activity. Tech that facilitates connectedness and sharing of results heightens those affordances. Tech activity using tech may involve a combo of routine mechanical actions driven by conceptual reasoning.