James Kaput and SimCalc MathWorlds

It feels reductive to seek to understand the professional life of James Kaput by looking at the software that he imagined, developed, and deployed. But, I have always believed that to understand the work of an individual, it is necessary to understand their life. And to understand their life, a close examination of the work, too, is required.

The work here is SimCalc. "SimCalc is a technology and curriculum research and development project intended to democratize access to the basic ideas underlying calculus beginning in the early grades and extending to AP calculus and beyond," (Kaput, 1999, p. 155). The operative concept here is *democratize access*. It is the defining feature and unifying thread that runs through all of Kaput's work. He was single minded in his pursuit of the use of multiple representations of mathematics to enrich student understanding, and the potential power the technology might have to bring those representations meaning as a tool to help all students see and understand formal advanced mathematical concepts. His purpose in the pursuit was to democratize mathematics and to create a gateway for all students into the previously elite ranks of the mathematically enabled and knowledgeable.

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Kaput completed his Bachelors in 1964 at Worcester Polytechnic Institute (Stickgold, 2005) and his PhD in Mathematics in 1968 at Clark University. He joined the faculty at the University of Massachusetts at Dartmouth in 1968 (Strickgold, 2005).

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Before being awarded the first major National Science Foundation grant that funded the SimCalc project in 1993, Kaput's interest in access was clear. The mode that his work would take to achieve this end was in presenting mathematical concepts with multiple representation. Before the manifestation of this concept was seen in the development of the software, Kaput used the form of multiple representations to present key concepts necessary for student understanding in the 1979 calculus textbook he published with Daniel Fleming (Hegedus, 2006; Tall, 2013).

Kaput worked and taught at University of Massachusetts Dartmouth for nearly his entire career (Strickgold, 2005). Part of the work he did there was to establish a program to help struggling freshmen learn enough mathematics in their first year in to open up the possibility for them to pursue careers that required high level mathematics. It is reasonable to assume that this experience affected the direction of his work for the rest of his life. At Dartmouth he often taught remedial college mathematics courses (Hamilton & Sabelli, 2013). He believed that if students could have access to advanced mathematical ideas early, it would negate the current role of advanced mathematics as gatekeeper. This experience had to have influenced his perspective on how students learn calculus.

Kaput believed that the key to providing access to all students to upper mathematics was not to wait until students had passed years of prerequisite coursework before giving students access to higher mathematical concepts, but to develop algebraic reasoning within the contexts of early elementary grade mathematics. His work in this area contributed to growing research on the capacity of students from diverse socioeconomic backgrounds to engage in algebraic reasoning (Hegedus, 2006). He loved his students, valuing them as human beings, supporting them and their personal growth. Though he worked in formal mathematics his focus was always on wider democratic and social issues (Tall, 2013). This perspective was evident in the development of SimCalc. It was clear that the software opened new ways to see the role that learners play in their own learning (Hamilton & Sabelli, 2013).

In conversation with Eric Hamilton about Kaput, Dick Lesh once recalled:

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He was always impressed by how Jim responded when he learned something new about how kids think or learn, or when he had to back out of one path and try another. There was a sense almost of giddiness that he had gone one step further in learning about kids thinking, and was a step closer to solving the problem of how to help kids navigate to success in acquiring and manipulating mathematics … he was more interested in learning how to create pathways that would be most productive." (Hamilton & Sabelli, 2013).

To provide access to all students, Kaput stated that the learning environment must provide "rich sets of actions that will expose underlying invariances and thus enable the student to heave a flexible and enduring web of mathematical meaning" (Kaput, 1989, p. 180). In order to introduce these advanced concepts to young children, the software that he developed provided an environment with integrated representations that included naturalistic animated images of swimming fish, runners, or elevators, along with interactive graphs and formal algorithms on the same screen so that the effects of changes in one form could be immediately see in the others. (citation).

Defining the difficulty. In a study with Judith Sims-Knight (Sims-Knight & Kaput, 1983), of 266 students who were nearing completion of the second year of algebra and 181 secondary students where were nearing completion of their first year of algebra, Kaput explored the ability of students to connect their formal mathematics with the wider world of experience. The students were given the following task:

> A farmer has found that over the years, for every 6 pigs he raises, he raises 1 cow. Let P stand for the number of pigs he raises and C stand for the number of cows he raises.

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Write an equation relating the number of pigs he raises to the number of cows he raises (p. 561).

To analyze the effect of the students' ability to imagine, Kaput and Sims-Knight conducted interviews of selected students to explore the nature of the students' imagery. Older students were more likely to use pictographic imagery to solve the problem, and were much more likely to solve the problem (9% vs. 29%). Kaput and Sims-Knight determined that student responses indicated that the difficulty in solving the problem lies in the disconnect between natural representations of concrete objects and the rules of algebraic symbol systems. The difficulty is in mapping from one system to another. The inability of the older students, who had had two years of experience with algebraic symbol systems, to complete the task indicated to Kaput and Sims-Knight that the translation skill must be explicitly taught if the mathematics that the students were learning could be usable in any significant way.

Using technology. Having looked at the effects on students of the inability to translate between different representations of the same mathematics phenomenon, Kaput explored the possible uses of technology in an article (1986) about emerging technology that linked simultaneous, multiple representations. Kaput stated that there were no computer-based curriculum at the time the article was written. In the article he reviews three mathematics software programs and considers the implications of the affordances that the software provided. His perspective on the software was that math is a tool of thought and communication that is essentially representational in nature, and that technology would have a profound impact on what it means to learn and use mathematics by providing access to new forms of representation.

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When discussing the representational nature of mathematics, Kaput (1986) was particularly interested in examining the point where the representation of mathematics systems and non-mathematics systems intersect. This is evidenced in the software that he chose to review in the article. The first software program that he reviewed allowed a user to use on-screen tools to create geometric diagrams and constructions. The central feature that he focused on was the ability for students to create a procedure that then could be executed by the student on other objects. He found the opportunity for student agency in this environment to be intriguing describing the student as both the source of knowledge generation and of questions. This environment, he commented, had multiple unnamed implications for teacher training (Kaput, 1986).

The second software program that Kaput reviewed presented a graph and the associated algebraic equation on the screen simultaneously. Changes that a student made on the equation could be seen instantly on the graph. Likewise changes that a student made on the graph would simultaneously change the values on the equation. As he discussed the nature of this second software program, he mused that a fuller environment could provide a means by which a student could predict effects across representations (Kaput, 1986): a capability that would eventually end up in SimCalc.

The final software program, still then in development, that Kaput reviewed in the article had the greatest impact on his SimCalc project. It too provided connections among different representations that were visible and useable simultaneously, this time using the concepts of ratio and proportion. What this software provided, in addition to a graphical representation on a coordinate plane and numerical representations, was a window where the student could build iconic representations. Kaput believed that the simple iconic

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representation simultaneously connected to the more sophisticated formal mathematical representations on the page would allow a student to build cognitive links that would be more tractable than any currently available. In addition, a form of multiple representation that reached from the primitive towards the abstract could be a tool that could create longitudinal coherence in the curriculum. The beginning-level representations increased the likelihood that the software environment would link with a student's existing representational schema. The simultaneous presentation of multiple general and abstract representations allowed for deeper mathematical commonalities and opened the possibility of introducing fundamental advanced mathematical concepts at earlier grade levels (Kaput, 1986).

Kaput concluded that software environments like the ones he reviewed would shape the direction of mathematics teaching and learning in the near future, given adequate teacher training, curriculum material support, and hardware availability. He imagined many tools including software that could enrich student understanding of multiplication using other bases, and algorithmic toolkits that would provide virtual manipulatives with corresponding formal representations. These, he predicted, will be the tools of the future (Kaput, 1986).

Handbook of Research on Mathematics Teaching and Learning

The exhaustive (at the time) *Handbook on Mathematics Teaching and Learning* was published in 1992, and James Kaput contributed the article titled *Technology and Mathematics Education* (1992). It is beyond the scope of this paper to summarize his significant contribution to the understanding of the relationship between mathematics education research and emerging technologies that covers 41 tightly printed double-columned pages. The presence of the article in the handbook speaks to Kaput's stature in the mathematics research community and the authority he held on the subject of technology and

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mathematics education research. For this paper, my interest in the article is in the way that the article foreshadowed his work on SimCalc.

I first read this article when I began my studies in mathematics education. It is a touchstone for mathematics education research and technology and, as of this writing, according to Google Scholar, has been cited more than a thousand times. Reading the article again within the context of Kaput's life and work, it is clear to me that he was thinking through the concepts and challenges that developing software for the teaching and learning of mathematics would bring. The most salient concepts within the article are the potential applications of computer capabilities and the impact that those applications could have on the very nature of learning and mathematical ideas.

Many of the concepts and concerns that Kaput approached in the previously discussed article (1986) are revisited in this article. Though in this article, Kaput (1992) places the concepts within their historical context. In addition he extends his thinking on the potential that the use of computers for the teaching and learning of mathematics brought to the classroom. Ultimately he considers what the impact of the changes that technology would bring would have on students in the future.

Computers up that that point had primarily been used as a convenient way to present traditional learning environments, acting as tutors to students, and leaving the students little to no agency within the learning environment. Contrary to the static software found in the current classrooms, Kaput envisioned leveraging the dynamic interactions that computer-based environments could provide. For example, he proposed the creation of a virtual environment that would store student procedures so that they might be replayed or manipulated to change outcomes (Kaput, 1992), similar to the software that allowed students

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to create and re-deploy geometric constructions in the previously discussed article (Kaput, 1986). The captured and stored procedures would allow for discussion between students and with teachers, and strategies could be developed. Temporal events could also be stored for later consideration. Compared this virtual possibility with the nature of physical structures (manipulatives), Kaput (1992) described the previous actions on manipulatives as "overwritten" once new actions were taken on them. In his view, inability to capture previous forms limited the usefulness of physical structures.

In addition to the ability to store the manipulations of virtual structures, Kaput (1992) discussed the potential for computers to present different notational systems simultaneously including between different mathematical structures, and between mathematical and non-mathematical structures. Manipulable simulations could model concrete terms while displaying formal generalizable abstract forms. At the same time, formal abstract forms could be displayed in concrete terms.

Within this dynamic environment, Kaput (1992) envisioned changes performed to be user-directed. The student-user would have control of the responses within each window, including refresh rates. This would allow students to predict the behavior of the environment before changes would be revealed. Student control of the links between graphs and character string notation would allow the student to expose different essential traits as they wish.

When Kaput discussed implementation of multiple-representational software, he addressed the need for teacher support by providing professional development for teachers to learn to use the software. He also discussed classroom management issues related to limited resources, shared computers, and time management. At the time the article was written one of the chief concerns was limited access to computers in schools (Kaput, 1992)

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SimCalc and the National Science Foundation

1993-1998. The initial grant by the National Science Foundation, "*SimCalc: Simulations for Calculus Learning*," was awarded in 1993. It was a continuing grant that ended in 1998 with a total award amount of $2,452,090 (National Science Foundation, 1993). The purpose of the project was to develop the conceptual and technological foundations for the development of curriculum for calculus as a mainstream strand starting with middle-school students and reaching into university precalculus and calculus classes. The purpose that would drive the SimCalc project through the entirety of its history is present in the abstract of the first grant. The stated purpose for the development of the curriculum was to democratize access for ALL (emphasis Kaput) students to the central ideas of calculus: movement, position, velocity. Explicit in the abstract is a refutation of calculus as a capstone course for the intellectual and professional elite. The abstract details how the researchers will integrate student daily experiences (the school bus trip home) and simulations into an understanding of motion using concrete and graphical representations.

During this time Kaput (1994) published an article that discussed the need to view calculus, not as the capstone course that it currently was, but as a web of ideas that could be taught from elementary school onward in a continuing coherent math curriculum. He traces the 200 year history of calculus as a province of the intellectual elite and discusses the role that dynamic graphical means for representing the ideas of calculus could reflect the origins of the study of change and motion.

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The mission of the project was also extended in an article published by Roschelle and Kaput (1996). At this time the description of the mathematics of SimCalc has been changed to the "the mathematics of change," (p. 97). The article includes screenshots of the software.

The name has been changed to SimCalc Math-Worlds, and the authors describe the environment as animated worlds that include dynamic links that reveal mathematical relationships. They discuss the need for students to engage in more sophisticated subject matter at younger age if they are to be prepared for the 21st century therefore, the target age group for the software is now 5 to 18 years of age, significantly earlier than the middle-school students described in the grant abstract. The authors describe the new Apple innovations of drag-and-drop and the ability to create interactive buttons. The article also states that Apple products were being used to develop teacher resources.

1997-2002. The second grant, "*SimCalc: Democratizing Access to the Mathematics of Change,*" was awarded in 1997. It was a continuing grant that ended in 2002 with a total award amount of $3,079,982 (National Science Foundation, 1997). As seen in the 1996 article (Roschelle & Kaput, 1996), though the concepts of calculus were still material to the content and purpose of the SimCalc software, the word "calculus" was no longer being used to classify the mathematical concepts addressed. The abstract describes the Mathematics of Change and Variation (MCV) as the concept that the project seeks to bring to all students. The researchers seek to expand their understanding of how technology can be used to expand on students' prior knowledge and expand MCV into traditional mathematics topics. At this point in the project the software is ready to deploy into teaching experiments in classrooms and community centers. The researchers will explore how the affordances of new technology can support the learning of "young, inner-city, minority children."

Kaput published several articles under this grant. Two in particular look at the importance of algebra with the lens that his current work provided. The first discussed the role that algebra had historically played as the institution, the constricted gateway (Kaput,

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1998). Algebra, the institution was the standard courses, texts, tests, and existing infrastructure that Kaput described as functioning as an engine of inequity. Kaput asserted that algebra could instead become an engine of mathematical power if students were introduced to the concepts as a web of knowledge and skill. Kaput argued that if students were introduced early to the practice of forming patterns and generalizing quantitative reasoning that they then would build deep and varied connections with all of mathematics. The coherence, depth, and power that this form of mathematics curriculum and the habits of mind that it would engender would allow for the elimination of the then current superficial mathematics of high-school algebra courses. The role of teachers and curriculum developers would be to start and integrate the language of algebra early, not through formalism but with ordinary language.

In the second article also discussed algebra's role as a gateway to higher mathematics (Kaput, 1999). In this article Kaput examined the overlapping and inter relational forms of algebraic reasoning. He summarized several qualitative studies that demonstrated how students created algebraic generalizations from interactions with concrete objects. In all of the studies students used natural language to discuss relationships. The youngest group of participants were in 2nd grade and the oldest were in the 6th grade. Topics covered included multiplication and functional behavior. In the final study 3rd and 4th grade students used SimCalc software to explore the movement of elevators. As in the other studies, students used natural language to build an understanding of motion. One thing to note is that throughout this article, while Kaput advocated for the introduction of algebraic concepts to very young students, he was clearly not advocating for the introduction of formal algebra.

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The purpose of the article was to provide examples of students building an understanding of algebraic concepts in *informal* ways.

2000-2004. The third grant, "*Understanding Classroom Interactions Among Diverse, Connected Classroom Technologies,*" was a continuing grant that ended in 2004 with a total award amount of $1,959,842 (National Science Foundation, 2000a). This research grant was for research that included corporate partners: Texas Instruments, Palm, and Nokia. The purpose was to explore the impact that the software would have on secondary (grades 8-12) classrooms if students interacted with hand-held devices. Teacher support was key as issues of classroom management and the impact that the connected classroom would have on student-teacher interactions were addressed.

At this time Kaput (2000) explored the impact that computers had on the long history of the representation of mathematics. Again discussing the implications of linked, editable, functions. The connection to the current grant is made as he discusses the use of TI-83 graphing calculators running in parallel in algebra and precalculus courses for academically weak first year college students. What he found was that when the software on the hand-held devices and the software on the computers ran in parallel, the students were able to flexibly switch between devices. Discourse analysis revealed that students were able to focus on mathematical objects and relations (Kaput, 2000). Learners become actively engaged through simulations, giving rise to learner agency. It was a way for learners to see themselves as doers. The mathematics was responsive to their actions. (Hamilton & Sabelli, 2013)

Kaput also explored the possibilities that a networked classroom, with students working on hand-held devices and the teacher working from a dedicated workstation, could provide. Students would be able to share functions of their own design or contribute to an emergent object. Targeted activities between students and the teacher, between students, or within or between small groups of students could also be developed. He expressed his expectation that the connected classroom would bring the same level of innovation and positive impact that computer connectivity brought to business and to the world-wide web (Kaput, 2000).

As previously stated, access was a core component of the push for the development of SimCalc. According to Eric Hamilton, at this time "Jim was on a moral mission, to make mathematics affordable and reachable. He never wavered from his conviction that the right vehicle was a mobile rather than desktop technology." (p. 468, Hamilton & Sabelli, 2013). The inclusion of the corporate partners makes sense in this light. The vested interest by leaders in the mobile technology of the time must have solidified the resolve of Kaput and his team.

An additional grant was awarded in 2000, "*Planning a Rigorous Experimental Trial of SimCalc's Approach to Increasing Access to Complex Mathematical Ideas."* It was a standard grant with an award amount of $116,123 (National Science Foundation, 2000b). The purpose of the grant was to prepare for the transition to a larger-scale, more rigorous testing of the SimCalc software. The targeted population for this grant was middle-school students. The software had already been tested in low performing districts in two states. Consistent with the ongoing mission of the project, the purpose continued to be to provide access to students from diverse populations. An exploration of what "high expectations" means for all students and the teacher support needed to reach those expectations is also included in the abstract.

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2002-2004. The follow-up grant to the planning grant was, "*Scaling Up SimCalc: Professional Development for Integrating Technology to Teach More Complex Mathematics (Phase 1)*," a standard grant with an award amount of $997,608 (National Science Foundation, 2002). Building on the evidence that SimCalc may improve learning outcomes, the purpose of the grant was to test the robustness of the software with teachers of varying backgrounds and content and technology knowledge. The intervention would include building on professional development that supports teaching with SimCalc, Geometer's Sketch Pad, and Fathom-data analysis.

In exploring the progress in developing a program that would democratize access to the Mathematics of Change and Variation (MCV), Hegedus and Kaput (2003), developed a 5-week after-school algebra enrichment program for middle and high-school students. The class met in a dedicated computer lab. The design of the study was quasi-experimental using a pre-post test methodology to measure student progress. The environment was a connected classroom using SimCalc software on hand-held calculators that connected to the teacher's computer. The test was a standardized algebra test developed for the state of Massachusetts. Hegedus and Kaput found that *all* students performed better relative to their prior knowledge.

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2004-2005. The final two grants awarded in 2004 to conclude Kaput's involvement in the SimCalc project are the scale up of the Phase I, "*Working With Teachers and Leveraging Technology to Scale Opportunities to Learn More Complex and Conceptually Difficult Middle School Mathematics,"* a continuing grant which ended in 2010 and had a total award amount of $6,463,414 (National Science Foundation, 2004a); and "*Representation, Participation and Teaching in Connected Classrooms*" a continuing grant which ended in 2009 and had a total award of $1,567,717 (National Science Foundation, 2004b). Both grants supported the continued rollout of SimCalc into the classroom. Even though both grants continued for several years, this paper will only look at work that James Kaput was directly involved with, which ended with his death in 2005.

A study funded under the first grant (Tatar, Roschelle, Knudsen, Shechtman, Kaput & Hopkins, 2008) looked at the effectiveness of SimCalc MathWorlds in a pilot experiment involving 21 seventh-grade mathematics teachers. What they found is that student gains did not depend upon long-term teacher professional development or a shift in pedagogy. The design was a controlled randomized experiment. Control teachers received no professional development beyond a basic foundation for teaching rate and proportionality (the topic used in the experiment). Treatment teachers were provided with training with SimCalc software and associated curriculum materials. A pretest-posttest design was used to measure students' learning gains before and after the units were taught. Teachers came from a wide range of situations including rural and urban environments. Teachers from the treatment group had greater gains in their mathematical content knowledge than teachers in the control groups. The most important outcome of the study according to the researchers was the growth in student mathematics knowledge. Students in the treatment condition gained significantly in knowledge of challenging mathematics compared to those in the control group. They did not learn more standard, formula-based mathematics. The gains held across the treatment teachers, indicating that SimCalc supported student learning under a variety of teachers and teaching contexts.

Under the second grant Kaput's long time colleague and friend Stephen Hegedus (2006) contributed an article dedicated to Kaput's life that sought to summarize his contributions to mathematics education research and tell the story of the creation of SimCalc.

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The article was one of the first that I read when beginning my research on James Kaput. It has been useful in giving me a cohesive overview of Kaput's work with the insight that only a close friend can offer. The contents of the article have been used throughout this paper.

Legacy

The impact of Jim Kaput's death on the mathematics research community appears to have been profound. It is clear from the many articles dedicated to him that he was beloved and respected by those who were fortunate enough to have worked with him (Hamilton & Sabelli, 2013; Hegedus, 2006; Tall, 2008). The response of the community was the creation of two remarkable volumes that trace his work and his legacy. The first was the June 2008 issue of "*Educational Studies in Mathematics*" (Hegedus & Lesh, 2008) which was dedicated entirely to Kaput's work.

The second volume is "*The SimCalc Vision and Contributions,*" (Hegedus & Roschelle, 2013). It is a monumental work that captures the contributions from the SimCalc research project and continues the conversation about the democratization of access to important mathematics, supported through dynamic representational software. It collects several important research studies related to the software. Sadly, it appears also to function as a capstone to the SimCalc project.

Jim Kaput did not foresee the development of smartphones and the mobile technology that have changed how we interact with each other, with information, and with the world. Though SimCalc was developed for handheld technologies that are not commonly connected in classrooms, his ideas of multiple representations are common in computer-based mathematics software. I have no doubt that if he were still with us he would be racing ahead, designing software for smartphones that would seek to provide access for ALL students.

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My Questions

I am curious about the many researchers that were highly invested in SimCalc. I wonder why they did not move the software toward development for the handheld devices that we currently use. Many researchers worked with Kaput over many years, yet the project, and research on the software seems to have faded.

When thinking about my own efforts to develop software that will provide access to students that are struggling, I think about what theoretical foundations that James Kaput used to support his work are applicable to mine. Though his 1992 article in the Handbook (Kaput, 1992) is still useful, in some ways it is already becoming an artifact from the past that did not envision the always-connected world that we currently inhabit. I wonder what groundbreaking change I will fail to see as I develop my software.

One of the later research studies (Tatar, Roschelle, Knudsen, Shechtman, Kaput & Hopkins, 2008) indicated that traditional mathematics classrooms (not reformed) produced outcomes using SimCalc software that were as strong as classrooms that were more constructivist in nature. This is an area of research that I think could be explored more fully.

What I have learned

I believe that the most valuable lesson I have taken from this exploration of the life and work of James Kaput is that an idea is not enough. Though his early ideas of the potential importance of simultaneously connected multiple representations were supported by research early on, he never stopped exploring theoretical constructs that dug deeply into how and why multiple representations might have real meaningful power in supporting student learning. Though I have not written of many of these explorations in this paper, I read numerous theoretical explorations while gathering information. A good many of those

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explorations have been saved and annotated for future use. The constant deep exploration for foundational support for his seemingly instinctual choices gave him a firm foundation to build upon when developing his software. Perhaps it was his training as a mathematician and his understanding of the role of proof that drove him to understand what could be argued and what could be accepted as tacit knowledge.

Kaput had a singular vision that held true throughout his work and beyond: the democratization of hard mathematics. I found it to be deeply meaningful that while he explored the possibilities that technology offered for learners, it was never the goal. It was always just a tool to help students connect to mathematical concepts and the richness that math can bring to the understanding of the world.

The last thing I have learned is that the world of mathematics research will continue after I am done. If I can build on the legacy in some small way that James Kaput left, I will be satisfied.

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Comments from Dr. H

Following the thread of the work via the grants awarded as artifacts of the trajectory is an interesting approach - particular for Kaput. You are correct that his focus was more on the design and implementation than the research. That he focused on pushing the field forward in thinking and challenging assumptions about the structure of algebra/calculus and its accessibility to students. His influence on the field is more in terms of his notions of how technology should be used to support the mathematics learning and how it would be used to represent foundational mathematical structures. This pushing forward ideas is important for shaping further research. While not referenced here, his collaboration with Maria Blanton on early algebraic thinking (e.g., elementary) is also significant. She was his colleague at UMass and is now at TERC.

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