Pilot Study Research Proposal

It is vital for prospective elementary teachers to acquire a solid foundation of mathematical knowledge in order to successfully navigate the path to the classroom and to successfully perform once they have entered their professional teaching life. Teacher training programs often require prospective teachers to demonstrate proficiency in mathematics by successful completion of mathematics content knowledge courses. In addition to the required coursework, professional licensure commonly requires candidates to pass tests of content knowledge Of course, the primary and ultimate goal of building mathematics content knowledge in prospective elementary teachers is to provide them with the tools and skills that they will need to support the acquisition of mathematical knowledge and skill in their future students (Ball, 1990; Fennema & Franke, 1992; Hill, Rowan, & Ball, 2005).

A longitudinal study (Ball, 1990) of 252 preservice teacher candidates at the point in which they entered formal teacher education found that many emerged from their content coursework with mathematics skills that were limited to discrete procedural processes disconnected from larger mathematical concepts. In addition, many of the prospective teachers focused on procedures and rules because they viewed the process of doing mathematics as simply following set procedures step-by-step to generate answers: that mathematics itself was an arbitrary collections of facts and rules to be remembered and employed. The author argued that the data suggested "that the mathematical understandings that prospective teachers bring are inadequate for teaching mathematics for understanding," (Ball, 1990, p. 464).

The purpose of this pilot study is to gain understanding of the learning processes, experiences, and depth of knowledge of undergraduate students in a mathematics content course designed for students with a designated elementary education emphasis. The study will explore the acquisition of procedural knowledge skills, specifically depth, fluency, and influence on conceptual knowledge. The research questions are:

- What are the experiences of students of varying achievement levels within a mathematics content course designed for undergraduate elementary education majors?
- In what way do undergraduate students in a mathematics content course for elementary education majors demonstrate the depth of their procedural knowledge gained throughout the course?

Literature Review

In order to study mathematics understanding and learning, a conceptual framework with definitions and meanings that are clearly understood within the mathematics research community must be established. Research can be designed around the framework, outcomes can be measured according to the operationalization of the components of the framework, and results can be understood within a shared conceptualization of the components. A well established and often used framework parses mathematical knowledge into two domains: procedural knowledge and conceptual knowledge.

The Conceptual and Procedural Knowledge Framework

The conceptual and procedural knowledge framework was defined by Hiebert and Lefevre (1986) in their seminal article to provide a useful way for researchers to understand student learning processes. They acknowledged that the relationship between procedural knowledge and conceptual knowledge was not yet well understood, and that often mathematics knowledge may be an inseparable combination of both forms of knowledge. In spite of this, they argued that distinguishing types of knowledge would provide a way to understand the failure or success of building mathematics understanding.

Hiebert and Lefevre (1986) define conceptual knowledge as a connected web of knowledge: a network of linked relationships that are as important as the discrete elements. The web of knowledge is conceptual only if the learner recognizes the relationships between elements. Conceptual knowledge not only describes the connections between known elements, it also describes the connections made between existing knowledge and newly acquired knowledge. The context is created by the network of relationships, allowing the learner the freedom to apply existing knowledge to novel problems.

Procedural knowledge as described by Hiebert and Lefevre (1986) is the use of the formal language and the symbolic representation system of mathematics. Algorithms or rules are used to complete tasks with an awareness of only surface features. The knowledge of the meaning of the processes is not accessed or necessary for successful completion of tasks. Step-by-step instructions prescribe how to complete tasks. These tasks may then be sequenced into superprocedures that incorporate lower level subprocedures. Procedural knowledge allows students to solve complex superprocedures as a chain of prescriptions without knowledge of the meaning of the task.

Three studies use the conceptual and procedural knowledge framework as defined by Hiebert and Lefevre (1986) to look at fractional knowledge in preservice teachers (Cheng-Yao, 2010, Lin et al, 2013; Rayner, Pitsolantis, & Osana, 2009). Fractional knowledge was investigated because of the complexity of the topic, and the difficulties that preservice teachers reportedly have had with knowledge of fractions. A comparison of the results across all three studies reveals procedural knowledge scores that are higher than conceptual knowledge scores. Using this framework, consistently procedural scores were higher than conceptual scores whether assessing for comparison to other groups (Lin et al, 2013; Cheng-Yao, 2010) or assessing for comparison to another variable (Rayner, Pitsolantis, & Osana, 2009). All three studies used purely quantitative analysis which did not allow for feedback by participants on their thinking processes as they solved problems. Consequently it is impossible to determine whether the participants made connections that were not demonstrated in the test results: whether they solved problems by rote processes or made choices driven by an underlying understanding of the concepts involved (Star, 2005).

Comparison of these studies. In this group of studies procedural knowledge was operationalized as the ability to solve algorithmic problems, and conceptual knowledge was demonstrated by the ability of the participants to verbalize their understanding. Two of the studies (Cheng-Yao, 2010; Rayner, Pitsolantis, & Osana, 2009) required the participants to demonstrate conceptual knowledge in a language rich environment. In their definition of conceptual knowledge Hiebert and Lefvre (1986) do not in any way define conceptual knowledge as the learner's ability to verbalize mathematical constructs. When discussing the difference between procedural and conceptual knowledge they describe story problems as conceptual and the number sentences as procedural, not as definitive, but as illustrative.

Verbal articulation of mathematical concepts may demonstrate conceptual understanding, but does the absence of a verbal response imply lack of conceptual understanding? The somewhat simplistic application of Hiebert and Lefevre's (1986) framework in these studies does not take into account the intersection and influence that one type of knowledge may have on the other. For all three studies the ability of the participant to solve equations was used only to assess procedural knowledge even when participants wrote explanations of their processes next to their solutions (Lin et. al., 2013). Though all three studies used pencil-and-paper tests, there was no provision to gather data on participants conceptual understanding that may have been embedded in their procedural work. When conceptual understanding was only tested by asking participants to create scenarios that would embody given mathematical expressions, conceptual theoretical mathematical understanding that may have been present but not applicable in a real-world context was not accessed or assessed.

The Relationship between Procedural Knowledge and Conceptual Knowledge

There is a tacit belief in the mathematics education research community that conceptual knowledge that can be articulated by the learner with normal language is a higher, more valuable form of knowledge than algorithmic procedural knowledge. This is position was not supported by Hiebert and Lefevre (1986). Though the oft cited article may be responsible for the familiar division of mathematical knowledge into two distinct types, the authors discuss at length the relationships between procedural knowledge and conceptual knowledge arguing that competence in mathematics relies on the "significant, fundamental relationships between conceptual and procedural knowledge." (paragraph 32).

In a review of the literature on procedural and conceptual knowledge (Rittle-Johnson, Schneider, & Star 2015), the authors explored the relationship between the two types of knowledge. They found that the broad agreement that conceptual knowledge supports procedural knowledge has resulted in ample research on the conceptual-then-procedural sequence of instruction. Consequently, though there is evidence that relationships between the two forms of knowledge are often bidirectional, there has been little research on the effect of presenting procedural instruction before teaching conceptual understanding. The authors conclude that "the belief that procedural knowledge does not support conceptual knowledge is a myth." (p. 594).

Three studies explored the effect of sequencing instruction between procedural skill and conceptual knowledge on the development of conceptual knowledge and procedural knowledge. The first (Rittle-Johnson & Alibali, 1999), a quasi-experimental study of fourth-grade students, presented compelling evidence of the possible iterative nature of the supportive interplay between conceptual knowledge and procedural knowledge. After completing a paper-and-pencil pretest, students were randomly assigned to either conceptual instruction, procedural instruction, or no instruction (control). The conceptual-instruction group instruction consisted of presentation of a problem and then being told that the amounts before the equals sign needs to equal the amount after it, meaning that the numbers need to add up to the same amount on both sides. No instruction was given on procedure.The procedural-instruction group was presented with a problem and the children were taught grouping procedures to solve. Two cycles of lessons followed by assessments were given to both instruction groups. A posttest of problems identical to the pretest was administered. Performance on transfer of knowledge to as a measure of conceptual knowledge was assessed by testing student performance on unfamiliar procedures.

Children who received instruction improved more than children who did not receive instruction. Most children from both instructions groups used correct procedures on the posttest. Procedurally instruction children used the instructed procedure on all posttest problem. Children from the conceptual-instruction group used multiple types of procedures, confirming that the conceptual instruction did not directly teach a specific procedure. Children who received procedural instruction increased their conceptual understanding. Children in the two treatment groups solved an equivalent number of problems correctly on the posttest. Children who were taught procedurally did not attempt to solve problems with minor variations in surface features. The authors state that these findings suggest that children may benefit most from conceptual instruction that helps them to invent correct procedures on their own. It is interesting to note that the conceptual instruction in this study was an explanation of the meaning of the equals sign. No real-world connections were made and all of the instruction that the children received was represented in the presented equation.

The second study was an experimental study that explored the effect of organizing arithmetic fact practice around equivalent practice (McNeil et al., 2012) on children ages seven through nine, examining the structure of early input and the role it may have in shaping children's understanding of fundamental mathematics concepts (operations on the left of the equals sign, equals sign means to *do* something). The authors chose to study the concept of mathematical equivalence because they state that it is "one of the most fundamental concepts in mathematics." (p. 11). The authors hypothesized that arithmetic practice that is organized by equivalent sums will lead children to construct a better understanding of math equivalence than practice that is not organized by equivalent sums.

The overall design was a posttest-only randomized experiment. Randomization occurred at the individual level with each child randomly assigned to the condition of equivalent values with practice problems grouped and presented by equivalent values, the condition of a shared addend with practice problems grouped iteratively by shared addend, or no extra practice over and above ordinary school homework. Three practice sessions one-on-one with a tutor were given with paper-and-pencil homework assignments between each session. A posttest was given to assess their understanding of equivalence and computation fluency. Children in the no practice condition were then assigned to either of the two practice conditions. This subset of children participated in a randomized experiment with a pretest, intervention, and follow up. Finally all children receive post-test assessment. The larger experiment was a posttest-only randomized experiment, with random assignment of individuals. The sub-experiment was a pretest-intervention-posttest design with random assignment of individuals.

The posttest assessment of all children to measure their understanding of math equivalence consisted of math equivalence equations, equation encoding by reconstructing math equivalence problems after viewing for a set period of time, and defining the equal sign by responding to a set of questions about the name and possible meanings for the equal sign. A follow-up assessment of the equivalence problems was given with the children instructed to make each side of the equal sign the same amount. If the child gave the correct answer the tutor gave positive feedback. If the child provided an incorrect number the tutor, using a script that emphasized the equal value of each side, provided the child with the correct answer. The purpose was to examine practice conditions on children's openness to learn from brief instruction.

Pretest scores were poor overall, with most children or all of the equations correctly. Children in the condition of equivalent values demonstrated better understanding in both problem solving and encoding performance than children in the other two conditions. Children in the equivalent values condition were more likely to define the equal sign relationally. The authors state that the results that organizing arithmetic facts into conceptually related groupings may improve children's understanding of mathematical equivalence. Performance on the follow-up assessment was still low with children in the equivalent values condition more likely than children in the iterative condition to solve correctly. Children in the pretest-intervention-follow up group who received the equivalent values practice were more likely than those who received the iterative practice to solve equations correctly. These children performed significantly better than the children who were not given the pretest. The authors state that this result suggests that practice organized by equivalent values may provide more benefit to children who have been previously exposed to math equivalence problems.

The third study was an an experimental mixed method study of 72 seven and eight year olds that investigated the effect of problem solving practice on learning (Canobi, 2009) and the interactions between conceptual and procedural knowledge. The study was designed to assess whether there is an iterative relationship between conceptual and procedural knowledge. Procedural knowledge is defined as the skills required to solve individual mathematical problems. The author states that exploring children's self-reported procedures with evidence of their problem-solving accuracy provide an accurate characterization of their procedural skills. Conceptual understanding is defined as knowledge about the underlying unifying principle of the structure of the problem. The author expected that highlighting conceptual relationships between problems through sequencing would increase procedural skill. Additionally, it was predicted that executing procedures to solve problems would improve children's report on conceptual relations between problems. It was anticipated that children's initial procedural skills would predict the level of conceptual advances made as a result of procedural practice.

The children were randomly assigned to either a conceptually sequenced condition, a randomly ordered condition, or a no-practice condition. Procedural and conceptual knowledge were tested before and after a practice phase. Participants were given a computer-based problem-solving task pretest of whole number addition and subtraction problems that allowed them to view the previous problem with the correct answer as they solved each problem. A second group of test problems consisted of children watching a puppet solve a problem using counters, and then responding to an interviewer about whether a the puppet could solve an additional problem using the same counters. After each problem the children were given feedback pointing out the nature of the relationship between the current problem and the previous problem. Children judged the commutative, subtraction complement, or identity relationship with the previous problem.

Practice was given randomly. Children received either conceptually sequenced worksheets, randomly ordered practice, or no practice as a control group. The sequenced problems were presented in conceptually sequenced pairs. This was expected to maximize opportunities to notice conceptual relations between problems as they executed problem-solving procedures. The randomly ordered practice condition was to provide equivalent practice with less obvious conceptual relationships. The no-practice control group completed nonmathematical worksheets. A posttest with identical procedures to the pretest was given. Children's reporting on conceptually sequenced problems was used as a conceptual measure and reporting on randomly order problems was used as a measure of procedural knowledge.

Procedural practice of randomly sequenced problems improved accuracy. Conceptual sequencing of practice problems enhanced children's ability to extend their procedural skills into new unpracticed problems. In addition, well-structured practice led to improvement in children's ability to identify and report on conceptual relationships between problems. As anticipated, the initial levels of procedural knowledge predicted the conceptual knowledge advances that the participants made. The author argues that these findings suggest that there is an iterative process between the development of both conceptual and procedural knowledge in children's addition and subtraction skills. The pairing of practice problems appeared to allow the participants to consider the relationship between the problems. The experience of solving practice problems that were ordered to make underlying concepts more obvious appeared to allow the children to apply their improved problem-solving skills to new unpracticed addition and subtraction problems. In comparison, the children who completed worksheets with randomly ordered problems only increased their scores on previously presented problems.

Children were able to verbalize key addition and subtraction concepts even though the intervention only involved asking the children to solve problems using their preferred problem-solving procedures. The author found that children's initial procedural skills influenced their ability to make conceptual inferences as a result of procedural practice (iterative cycle). The author argues that when children know answers to problems they have a greater capacity to notice the principles that link the problems together, especially when the problems are presented in ways that make the links overt. Because these children do not need to focus on computational requirements, they are able to make discoveries about the structure of the domain. This study was limited to young children and addition and subtraction processes.

Comparison of these studies. The three preceding studies defined conceptual knowledge as an understanding of the underlying mathematical principles present in presented equations. The authors of the first **(**Rittle-Johnson & Alibali, 1999) and third (Canobi, 2009) studies describe the need to reach beyond the effect that procedural and conceptual knowledge have on each other into an exploration of how the effect occurs. While the evidence on the iterative nature of the two forms of knowledge is compelling in these studies of children, it would be useful to determine the nature of their interaction in other populations.

While I previously stated that Hiebert and Lefevre (1986) do not define conceptual knowledge as verbal knowledge, they do argue that for procedural knowledge to include conceptual knowledge it must contain within it, for the learner, a connection to the real world. In this regard, the operationalization of conceptual knowledge in these three studies (Canobi, 2009;

McNeil et al., 2012; Rittle-Johnson & Alibali, 1999) was a complete departure from Hiebert and Lefevre's conceptual and procedural knowledge framework. In all three studies conceptual knowledge was measured as an understanding of the mathematical concepts represented by procedural work. From this vantage point the three studies are were fact completely situated in the procedural knowledge realm, and may in fact be studies of deep procedural knowledge (Star, 2005).

Knowledge Type and Quality

In a response to Hiebert and Lefevre's (1986) bifurcation of mathematical knowledge into conceptual and procedural types, Star (2005) redefined conceptual knowledge as knowledge of concepts, principles, and definitions; and procedural knowledge as knowledge of procedures, algorithms, and the sequence of steps used in problem solving. He argued that contrary to the common perception of procedural knowledge as superficial and rote and the perception that conceptual knowledge is knowledge that is known deeply, that both types of knowledge can be either superficial or deep or anything in between. He described the depth of knowledge as *knowledge quality*.

Using this alteration of the conceptual/procedural framework, the following two studies focus on flexibility in procedural knowledge. Researchers in both studies interpret flexibility in procedural processes, or knowledge of multiple strategies, as an indication of deep procedural knowledge. Looking at procedural knowledge without reference to conceptual acknowledges the importance of procedural knowledge as valuable in and of itself (Star, 2005). The participants of the first of these two studies were undergraduates. The participants in the second study were grade school students.

The first of these two studies (Maciejewski & Star, 2016) was a teaching intervention designed to promote flexibility in procedural knowledge in first year undergraduate calculus students. The researchers sought to determine not only if procedural flexibility could be developed, but also if it resembled expert-like procedural performance. The design was quasi-experimental pretest/post test. Two sections of an introductory calculus course taught by the same instructor were selected for the experimental study. A pretest on differentiation was given. After receiving a lesson that contained a traditional sequence of instruction on procedures, the control section was given a typical worksheet for homework. The treatment section was given a worksheet that specified two approaches for solving each assigned problem and were asked to describe which method they prefered. The homework assignment was followed by a readministration of the pretest as a post test.

An analysis of the data found that here were no significant differences in the sections' score averages. Both groups scores improved on the posttest. The treatment group used a greater variety of strategies than the control group. The authors found that many of students chose to use processes that took longer to solve possibly because of familiarity with the form. Even though some students chose to use longer processes, the researchers determined that the treatment group moved closer to expert-like performance. The researchers felt that as novices the students had not yet developed adequate problem classification schema and that they would become more efficient with more experience.

The authors concluded that it is possible to use an instructional task to support the development of undergraduate students' flexible use of procedures. Because the control group did not demonstrate flexibility after practice, the authors determined that an activity that prompted critical reflection by presenting tasks that prompt students to resolve questions in different ways and allow for the comparison of different solutions may support the development of deep procedural knowledge.

The second study (Lamb, Bishop, Philipp, Whitacre, & Schappelle, 2016) used clinical interviews to investigate the relationship between student flexibility in procedural problem solving and mathematics performance in students grades 2, 4, 7, and 11. The researchers sought to determine the degree to which flexible ways of reasoning influenced performance on integer problems. The wide grade span was chosen to cover a wide range of student learning experiences: from those who had not yet received school-based integer instruction to students who were enrolled in precalculus or calculus courses and therefore deemed to be successful high school mathematics students.

Individual clinical interviews were conducted and videotaped at the students' school sites. The interviews were standardized and all students were asked to complete the same 25 open number sentences. Interviews were coded both for underlying reasoning and for correctness. Five broad categories and 41 subcodes provided detail into student's specific strategies. The five categories broken down into ways of reasoning: order-based, analogy-based, computational, formal, and developmental. Flexibility was measured by the variety of ways that students used to solve integer-arithmetic tasks. Proficiency with a particular way of reasoning was demonstrated when a student used it three or more times. The number of ways that students used ways of reasoning that they were proficient in was the measure of flexibility.

Case studies were performed on three 7th grade students who exemplified the relationship between flexibility and accuracy. The first student chosen completed 32% of the open number sentences correctly. The second student completed 64% of the problems correctly. The third case completed 100% of the problems correctly. The researchers had found that the seventh grade students had the greatest spread in flexibility. Eighty-five percent of 11th graders had used 3 or 4 methods. Flexibility scores had correlated positively with performance. This result also held across the case studies. The first and second cases used one type of reasoning almost exclusively which limited their options for solving problems. Though each of them focused on *different* forms of reasoning, the fact that each student presented one way of reasoning appeared to negatively influence success. The third case, who had completed every open number sentence correctly, flexibly used a wide range of strategies on the problems and appeared to choose strategies that corresponded with the underlying structure of the sentence.

Case studies provided insight into the relationship between flexibility and performance on open number sentence problems. Across all age groups the correlation between flexibility and performance held. The authors concluded that students who rely on a single way of reasoning may be impeded in their success because of their limited flexibility and that multiple ways of reasoning promotes successful performance. For every participant group the correlation between flexibility and accuracy held; more flexible students were more successful.

Comparison of these studies. Though both studies (Lamb et. al., 2016; Maciejewski & Star, 2016) addressed flexibility in procedural knowledge, the approaches were very different. Though neither study used the language of the conceptual and procedural knowledge framework, both studies explored the impact of deep procedural knowledge (Star, 2005) exemplified by procedural flexibility. Depth of conceptual knowledge was implied in the Maciejewski and Star (2016) study within the discussion of students' process of which method to use to solve problems for those students who used multiple methods. In the Lamb et. al. (2016) study both the clinical interviews and the more in-depth case study interviews provided insight into the procedural and conceptual understanding of the participants. Participant responses allowed the researchers to determine not only the presence of reasoning types but also the manner in which they were chosen. The interview process uncovered participants' procedural and conceptual knowledge, whether solutions were pursued through rote processes or driven by connected knowledge (Hiebert and Lefevre, 1986) and the depth or lack of depth of the participants' procedural and conceptual knowledge (Star, 2005).

The Exploration of Knowledge Embedded in Mathematical Symbols

The studies that explored the iterative relationship between conceptual and procedural knowledge (Canobi, 2009; McNeil et al., 2012; Rittle-Johnson & Alibali, 1999) and procedural flexibility (Lamb et. al., 2016; Maciejewski & Star, 2016) examined participants' conceptual knowledge by analyzing their responses to the symbolic representation of mathematical constructs. Across these studies conceptual knowledge was seen as understanding of the underlying mathematical constructs represented by equations and mathematical expressions. Whether assessing procedural or conceptual knowledge, the focus was on the symbolic representations. The position of these studies was that student work traditionally described as procedural (Heibert and Lefevre, 1986) potentially held conceptual understanding that manifested through transfer of knowledge to novel problems **(**Canobi, 2009; Rittle-Johnson & Alibali, 1999), or description of the underlying meaning of mathematical symbols (Canobi, 2009; Lamb, et. al., 2016; McNeil et al., 2012)

Pilot Study Design, Methods, and Procedures

For the purposes of this study, procedural knowledge will be defined using Star's (2005) framework of knowledge type and quality. Conceptual knowledge will be defined using Heibert and Lefevre's (1986) description of the relationships between existing knowledge and new knowledge and the ability to apply existing knowledge to novel problems. The focus will be primarily on the development of procedural knowledge and the impact of that knowledge on the acquisition of further knowledge and conceptual understanding by undergraduate students in an elementary education mathematics content course.

If the primary strength of undergraduate mathematics knowledge is in procedural knowledge, then it makes sense to use the existing procedural knowledge as a tool to build conceptual understanding. Rather than perceiving shallow procedural knowledge of undergraduate students as a liability, perhaps it is simply the starting point, the building blocks, that can be used to create deep procedural knowledge by providing the opportunity for students to build the net of knowledge that interconnects the disparate parts of their existing mathematics understanding. Only a fragmented picture of students' mathematical knowledge can be acquired using pretests, surveys, or interviews. Conducting an ethnography of this population as a participant-observer over the course of a semester will allow me to assemble a more complete understanding of their challenges, experiences, and successes relative to their procedural knowledge of mathematics.

Research design and data collection.

Ethnographic studies investigate cultures by examining interpersonal, social, and cultural aspects of the participants using ethnographic interviews and participant observation to gather data. There are three distinct elements of an ethnographic study. The first is the use of ethnographic research methods to gather the data, the second is the resulting data gathered as a participant observer, and the third is the interpretation of the data gathered (Shagrir, 2017).

I will use Wolcott's (2008) description of ethnographic research as *experiencing*, *inquiring*, and *examining* as the organizing framework for data collection. *Experiencing* will be be accomplished by being present in the classroom to observe the day-to-day activities of the students. I will generate field notes of interactions between members of the class and record details that I see and hear (Wolcott, 2008). As a participant observer, my intent is to take an essentially passive role. Wolcott (2008) advised that the participant observer should become "only as involved as is necessary to obtain the information desired" (p. 49). *Inquiring* will take place through casual conversation, semi-structured and unstructured interviews, and questioning to clarify student understanding when opportunities arise. In addition to the observations gathered through experiencing, I will use field notes to record interactions with students and their comments and responses to clarifying questions. Responses to interviews will provide additional data. *Examining* will consist of an examination of the work generated by the students during the course, including student homework, student tests, and other student generated items.

Data analysis. Data analysis will use the constant comparative method (Eisenhart, 1988). Analysis will occur throughout the period of study as data is gathered. Triangulation will be used to compare categories and relationships across data. Theoretical explanations will develop through a recursive process of examination to develop an understanding that encompasses all of the data and provides a comprehensive picture of meanings of the students' experience.

Personal Biases. I will examine and explore my personal biases throughout this study. My beliefs about the role of procedural knowledge math student learning processes will affect my perspective on the data that I gather for this study. In order to proceed in the most objective way possible, I will examine my beliefs against the data that support or challenge those beliefs as a regular portion of my field notes.

Limitations. As a single observer in the class, data collected through observation will be limited to my perspective alone. The class that will be observed is credit bearing for the students and it is imperative that research activities do not negatively affect the achievement of students within the course. This circumstance may have unforeseen effects on my ability to gather data.

Importance of the study. This study will add to the existing literature on elementary education undergraduates' experiences in mathematics content courses, and will add to the body of work on research on the development of procedural knowledge. There is an identified need for in-depth studies of teachers as learners of mathematics (Mewborn, 2001). Star commented on the scarcity of research of procedural knowledge (2005). To my knowledge, there does not exist an ethnographic research study that examines the experience of undergraduates procedural learning within an elementary education mathematics content course.

References

Ball, D. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466

Canobi, K. H. (2009). Concept–procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology, 102*(2), 131-149.

Cheng-Yao, L. (2010). Web-Based Instruction on Preservice Teachers Knowledge of Fraction Operations. *School Science & Mathematics, 110*(2), 59-70.

Eisenhart, M. A. (1988). The ethnographic research tradition and mathematics education research. *Journal for Research in Mathematics Education, 19*(2), 99–114.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 147-164). New York: Macmillan.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics* (pp. 1-27). Hillsdale, NJ: Erlbaum.

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Lamb, L., Bishop, J., Philipp, R., Whitacre, I., & Schappelle, B. (2016, November). The relationship between flexibility and student performance on open number sentences with integers. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.) *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 171-178).

Lin, C., Becker, J., Byun, M., Yang, D., & Huang, T. (2013). Preservice Teachers' Conceptual and Procedural Knowledge of Fraction Operations: A Comparative Study of the United States and Taiwan. *School Science & Mathematics*, *113*(1), 41-51.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Education Research Journal*, *3*, 28-36.

Maciejewski, W., & Star, J. R. (2016). Developing flexible procedural knowledge in undergraduate calculus. *Research in Mathematics Education*, *18*(3), 299-316.

McNeil, N. M., Chesney, D. L., Matthews, P. G., Fyfe, E. R., Peterson, L. A. Dunwiddle, A. E., & Wheeler, M. C. (2012). It pays to be organized: Organizing arithmetic practice around equivalent values facilitates understanding of math equivalence. *Journal of educational psychology, 104* (4), 1109 - 1121.

Rayner, V., Pitsolantis, N., & Osana, H. (2009). Mathematics anxiety in preservice teachers- Its relationship to their conceptual and procedural knowledge of fractions. *Mathematics Education Research Journal*, *21*(3), 60-85.

Rittle-Johnson, B, & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, *9*(1), 175-189.

Rittle-Johnson, B., Schneider, M., & Star, J. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. Educational Psychology Review, 27(4), 587–597. https://doi.org/10.1007/s10648-015-9302-x

Shagrir, L. (2017). *Journey to ethnographic research*. Cham, Switzerland: Springer Nature

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, *36*, 404–411.

Wolcott (2008). *Ethnography: A Way of Seeing*. Lanham, MD: AltaMira Press.