

Synthesis of Research on Procedural Knowledge

Much of the research on procedural knowledge defines the concept by comparing and contrasting it to conceptual knowledge. Hiebert and Lefevre (1986) define procedural knowledge as *how* and *when*, and conceptual knowledge as *why*. Star (2005) interprets this approach as defining procedural knowledge as shallow (route manipulation) and conceptual knowledge as deep (connected and interpretive).

This is the approach that was taken in the first three studies I reviewed (Lin, Becker, Byun, Yang, & Huang, 2013; Rayner, Pitsolantis, & Osana, 2009; and Cheng-Yao, 2010). All three studies looked at procedural and conceptual knowledge of fractions in preservice teachers.

Study 1

Purpose. The purpose of the first study (Lin, Becker, Byun, Yang, & Huang, 2013) was to examine differences in preservice teachers' procedural knowledge and conceptual knowledge of fraction operations in China (Taiwan) and in the United States.

Methods. The design was a quantitative study using a paper-and-pencil test adapted from a previously validated instrument developed to assess 4th and 5th grade students (Cramer, Post, and del Mas, 2002) to determine teacher procedural and conceptual knowledge of fractions. The test covered addition, subtraction, multiplication, and division of fractions. Procedural knowledge was tested with algorithmic questions. For conceptual knowledge, participants were asked to create models for the algorithms that could be used for teaching children. The paper-and-pencil test allowed participants to explain their thinking, and researchers to standardize grading. The test was administered the first week of class in the fall to 47 preservice teachers in Taiwan and 47 preservice teachers in the United States.

Results. Comparing the fractional knowledge of Chinese and Americans, the researchers found that overall the Chinese teachers did significantly better than the Americans on all procedural items, and on all conceptual dimensions except division which displayed lower scores for teachers from both countries. There was a weak, but not statistically significant, positive correlation between procedural and conceptual knowledge in addition, subtraction, multiplication, and division of fractions of teachers in both the United States and Taiwan. Findings were similar to several named studies in the article. Procedural knowledge scores exceeded conceptual scores in all dimensions for teachers from both countries.

Critical Comments. The researchers determined that the weak correlation between conceptual and procedural knowledge of fractions suggests that higher procedural knowledge does not cause nor predict higher-level conceptual knowledge. Implications for teacher education included the need to develop content knowledge to avoid the poor teaching that comes from a teacher who is insecure in subject knowledge. The researchers also state that though prior international comparisons of mathematics achievement has been focused on grades 4-12, comparative analysis of preservice teacher performance is essential, allowing comparisons between countries and the opportunity for researchers and teachers to learn from each other.

Limitations of the study included sample size (97 subjects), that only preservice teaching majors were included, and that the study only covered knowledge of fractions.

Study 2

Purpose. Exploring the role of anxiety in the interplay between knowledge and practice, the purpose of the second study (Rayner, Pitsolantis, & Osana, 2009) was to analyze

the possible relationship between procedural and conceptual knowledge of fractions and mathematics anxiety in preservice teachers. Researchers conjectured that high levels of procedural instruction may have a role in increasing anxiety, and that an increase in conceptual understanding result in a decrease in anxiety.

Methods. Thirty-two undergraduate preservice teachers knowledge of fractions were assessed using a validated paper-and-pencil test designed to assess upper elementary students' procedural and conceptual knowledge of fractions (Saxe et al., 2001) with an additional 2 researcher-developed items to test conceptual knowledge. Researchers used the Revised Mathematics Anxiety Rating Scale (RMARS, Baloglu, 2002) developed for undergraduate and graduate students to measure mathematics anxiety in the participants. The resulting data allowed for a quantitative analysis to determine the relationship between knowledge of fractions and mathematics anxiety.

Results. The researchers found that preservice teachers scored higher on procedural knowledge than conceptual knowledge. Seventy-two percent of the participants scored in the middle range for anxiety, 19% were in the low range, and 10% were in the high range. When comparing knowledge scores with anxiety results, researchers found that there was a significant negative correlation between both procedural and conceptual knowledge and anxiety. Greater anxiety correlated with fewer correct responses.

Critical comments. This study replicated the negative relationship between mathematics anxiety and performance on complex procedures found in previous studies of undergraduate psychology students. The study adds to the body of literature on the relationship between content knowledge and mathematics anxiety and is the first to examine the role of mathematics understanding in preservice teacher anxiety. The researchers

surmised that the negative correlation between conceptual knowledge and anxiety is consistent with previous studies and may indicate that conceptually based instruction facilitates a more meaningful understanding of mathematics. The researchers determined that procedural proficiency should also be addressed.

Limitations included small sample size (32 participants), limited scope (fractions only). The design of the study did not allow for the exploration of causal relationships.

Study 3

Purpose. The purpose of the third study (Cheng-Yao, 2010) was to determine whether web-based instruction is a more effective method for preservice teachers to learning procedural and conceptual knowledge of fractions than traditional instruction.

used tests of preservice teachers' procedural and conceptual knowledge of fractions to determine

Methods. The study was a pretest/posttest experimental design with forty-eight elementary education math methods students randomly assigned to complete a 6-week fractions unit delivered by the same instructor with either traditional instruction or web-based instruction. The instrument used to assess knowledge was adapted from Cramer et al. (2002) and Ma (1999) modified by the researcher to provide more emphasis on procedural and conceptual knowledge. A quantitative analysis was conducted on the test scores.

Results. The pretest scores for procedural knowledge were higher than conceptual knowledge. Using the definitions drawn from Hiebert and Lefevre (1986) the researcher determined that the results indicated that participants could *do*, but could not explain *why*. Post test scores on web-based instruction were significantly higher than for traditional

instruction. Conceptual scores were also significantly higher for online class instruction than for traditional instruction but remained lower than procedural scores.

Critical Comments. Results suggest that web-based instruction is efficient as indicated in earlier studies. This may be because of the student-centered approach, room for student exploration, and immediate feedback.

Limitations included the short time span covered, limited subject content (fractions), small sample size (48 participants), and inclusion of only elementary education majors. These factors limit generalizability of the results.

Reflection on Studies 1-3

In reviewing research on procedural knowledge of preservice teachers I did not specifically seek out studies of fractional knowledge. All three studies chose to look at the topic because of the importance of fractional knowledge, the complexity of the topic, and difficulties that preservice teachers reportedly have had with knowledge of fractions. The similarities in the results of these studies invites a comparison of the results that is beyond the scope of this paper. A cursory reading of the results across all three studies reveals procedural knowledge scores that are higher than conceptual knowledge scores using the lens that the researchers chose to define procedural knowledge. All three studies used purely quantitative analysis that did not allow for feedback by participants on their thinking processes as they solved problems. Consequently it is impossible to determine whether the participants shallow or deep: whether they solved by rote processes or made choices driven by an underlying understanding of the concepts involved (Star, 2005).

Studies 4 and 5

Rather than comparing and contrasting results of assessments of procedural and conceptual knowledge, the following two studies focus on flexibility in procedural knowledge. Researchers in both studies interpret flexibility in procedural processes, or knowledge of multiple strategies, as an indication of deep procedural knowledge (Star, 2005). As in the prior three studies, the participants of the 4th study were undergraduates. The participants in the 5th study were grade school students.

Study 4

Purpose. The 4th study (Maciejewski & Star, 2016) was a teaching intervention designed to promote flexibility in procedural knowledge in first year undergraduate calculus students. The researchers sought to determine not only if procedural flexibility could be developed, but also if it resembled expert-like procedural performance.

Methods. The design was quasi-experimental pretest/post test. Two sections of an introductory calculus course taught by the same instructor were selected for the experimental study. A pretest on differentiation was given. After receiving learning a typical learning sequence, the control section was given typical worksheet for homework. The treatment section was given a worksheet that specified two approaches for solving each assigned problem and were asked to describe which method they preferred. The homework assignment was followed by a re-administration of the pretest as a post test.

Results. There were no significant differences in the sections' score averages. Both groups scores improved on the post test. The treatment group used a greater variety of strategies than the control group. Many students chose to use processes that took longer to solve because of familiarity with the form.

Critical comments. Even though some students chose to use longer processes, the researchers determined that the treatment group moved closer to expert-like performance. The researchers felt that as novices the students had not yet developed adequate problem classification schema and that they would become more efficient with more experience.

A limitation of the study is that researchers did not explore the socio-mathematical norms of the classroom and there is a chance that student choices were driven by a perception of teacher expectation. Also, students were not randomly assigned to the treatment group, though the researchers determined that the two sections were fairly homogeneous.

Study 5

Purpose. The 5th study (Lamb, Bishop, Philipp, Whitacre, & Schappelle, 2016) investigated the relationship between student flexibility in procedural problem solving and mathematics performance.

Methods. The researchers used their Ways of Reasoning coding scheme to analyze 60-90 minute clinical interviews of 160 students grades 2, 4, 7, and 11 (40 from each) solving integer tasks. Flexibility was defined as a measure of the variety of ways that students approached solving 25 open number sentences. Interviews were coded both for underlying reasoning and for correctness. Case studies were performed on three 7th grade students who exemplified the relationship between flexibility and accuracy.

Results. Seventh graders had the greatest spread in flexibility. Eighty-five percent of 11th graders used 3 or 4 methods. Flexibility scores correlated positively with performance. This result held across the case studies with the students who had 32% and 64% correct using only one form of reasoning, and the student who had 100% correct using a wide range of strategies that corresponded with the underlying structure of each problem.

Critical comments. Case studies provided insight into the relationship between flexibility and performance on open number sentence problems. Across all age groups the correlation between flexibility and performance held.

Reflection on Studies 4 and 5

Though both studies 4 and 5 addressed flexibility in procedural knowledge, the approaches were very different. Both studies cite Star (2005) as they define deep procedural knowledge. The populations and the topics studied were varied, as were the methods used to analyse the data. Though Star's article (2005) is not recent, I have found little recent research on deep procedural knowledge. There remains a wide landscape to be explored and understood.

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